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Describing the Human Economy

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Maria Augustinovics

INTRODUCTION

This paper is not a progress report, it is rather a bit of wishful thinking on future possibilities. It is built upon many pieces of my previous work in various fields, and in this sense it is a kind of summation. The end result is the opening up of vast new fields that would be good to explore in the coming years.

In Chapter 1 I join those fellow economists who believe that the conventional scope of economic theory is too narrow. I will argue that economics as a science should be able to explain all segments of human economic activity, including non-market (or subsistence, or household) economy as well as the economic aspects of human life itself. I will also argue that a proper description and analysis of observable facts must precede the formulation of assumptions and theories and that a number of methodological requirements must be met for this purpose.

In Chapter 2 I try to outline the skeleton of a descriptive framework which would satisfy some--but not all--of those requirements. In principle, such a framework is meant to serve

as a background catalogue to define the scope--assumptions, abstractions, neglect--of individual, theoretical or practical economic models properly. In the way of wishful thinking, however, I should not like to exclude the possibility of some analytical studies within a similar, but more advanced, framework.

In Chapter 3 I turn to the more formal aspects of description pointing to a few of the simplest mathematical tools that could contribute to our understanding of indirect, circular interdependencies within the economy.

Notes, tables and figures are collected at the end of the paper, the usual result of writing in a hurry and editing the material for the first time, but an inconvenience to the reader, for which I apologize.

1. THEORY AND METHODOLOGY

1.1 The Narrow Economics of the Market

Economics has always been fascinated by the market. Rightly so, since market is the most amazing product of human economic activity. It has become a power beyond and above human will, it has made economy the blind master and man its defenceless servant. It helped to create the modern nation and it is now in the process of creating a supranational, although not truly international, system of forces beyond and above national will. However, there are many different ways of looking at the market.

You may remain obsessed with the market place, watching as individual agents go about their various transactions there and wondering what they think they are doing.[1] You may assume that they all have perfect information, they are all perfeclty competitive, they all behave perfectly rationally in their own well-known interests, and you will find that this brings about the most perfect well-being for all concerned. And, naturally, this has been going on since time immemorial and will have to go on forever. This kind of approach, with due respect and apologies to the more sophisticated formulations of the theory, is then called "neo-classical economics", "general equilibrium economics", "microeconomics", "mainstream" or "mainline economics", etc. In spite of all that has already been said and written against it, this approach is amazingly persistent. Partly because it is convenient to maintain innocence: what you do not know about society and social power cannot hurt you; partly because it has incorporated a huge vested interest by first-rate mathematical brains. This on its part lends a poor economic theory such elegance, such intellectual rigor[2] which is certainly attractive to the demanding student.

You may take a broader, and therefore a more convincing, view of the market. You may realize that no matter what individual agents think they are doing it is the aggregate output, the aggregate supply and demand that count. You may also realize that in the world of "exchangeables" there are such distinguished things as money and labour, that a society has to worry about growth and recession, about inflation and employment, that market alone does not take care of everything but that some purposeful social intervention is required, whether you like it or not. This approach is then called "macroeconomics", or "Keynesian economics", it has often been called "the Keynesian revolution".[3] Contemporary theory belonging to or originating from this school, called "neo-Keynesian" or "post-Keynesian" economics, has made further important steps towards economic realities--it is a pity that eminent representatives of it are so emotionally anti-mathematical, as if mathematical tools would be responsible for nonsense in economics.[4]

Or, you may be looking at the market and try to discover what is going on beyond it, to understand its historically determined place and function within the economy as a whole, within the economy defined as the mode of interaction between man and nature. Then you will realize that the market is an important link in the circular chain of production-distribution-consumption, in the process of reproduction. It is the part of the system that makes interaction and thus division of labour possible among socio-economic units separated by ownership. In other words, it is the ingenious device that turns individual work into work for others and increasingly for the whole society under the conditions of private property. Division of labour then provides for increasing productivity, leads to the accumulation and concentration of the means of production, of wealth and social power. This kind of approach to the market was adopted by what is called "classical economics" or "political economy"; in its last and most c nsistent form, the "Marxian economics". There is probably no need now to say that the author of this paper finds this approach the most enlightening and therefore superior to the others.

The classical, and particularly the Marxian approach, has been ignored or neglected, laid *ad acta* or buried so many times by so many over the past hundred years that one should be surprised how very alive it is. Moreover, the approach--may be without some of the specific notions, with different conclusions and dressed in different language--is being increasingly adopted as a starting point by economists who do not consider themselves to be Marxists but who seriously attempt to understand the realities of economy and society.

Nevertheless, there is no reason for celebration in this camp either. Ignoring the arrogant remarks made by ignorants who never took the trouble of reading Marx or never got beyond the first volume of *Das Kapital* one has to admit that much of the serious and honest critique is justified: relatively little has been achieved in bringing Marxian economics over from the late 19th century economic reality to the late 20th century economic reality.

The reasons are numerous[5)], so are the new phenomena not properly investigated and explained by contemporary Marxian economics. The present paper, however, is not about economic theory in general. There is only one point to be made here, a point where unfortunately contemporary Marxism is as negligent as any other school of economic thought. Namely, no matter what approach we adopt when looking at the market, we usually do not look at those layers of human economic activity that have not been--so far, or will never be--absorbed by the market.

For neo-classical economics market has always existed. For the classical economists and for Marx, market was a historical achievement. Observing its rapid penetration into the traditional subsistence economy they took it for granted that this process will once be fully completed, that the economy will soon be transformed into a pure capitalist market economy. For them subsistence or pre-market economy was a matter of the past, a matter of ancient or Asian modes of production.

The assumption of pure capitalist market economy directly leads to the assumption of pure socialism. Since if the

historic task of the capitalist market had been performed, if the whole reproduction process has reached the ultimate level of concentration and centralization based on social distribution of labour, then only one brave revolutionary action is needed: to abolish private property that has become obsolete and to replace the market, that has lost its historic function by purposeful social control of the economy. Much--may be even most--of the problems of existing socialist societies can be traced back to this assumption that was applied in circumstances where the historic task of the capitalist market was far from being completed.

In fact, economic activity has not been completely absorbed by the market, at least not until the end of the 20th century. While it still remains the main trend for the market to expand, to penetrate deeper and deeper into traditionally subsistence layers of the economy, powerful reverse tendencies can also be observed for various economic and social reasons. The existence of non-market economic activity is a fact in capitalist and in socialist economies, not to speak of the so called Third World. It is also a fact ignored by all major schools of economic theory.

What may be worse is that non-market economic activity is also neglected, with some inconsistent compromises, in the empirical evidence available. The System of National Accounts, a remarkable achievement of recent decades, has accepted the theoretical assumptions and the division of economic units into "firms" (further subdivided) and "households", a division which goes with the underlying theoretical assumption and which is a trap. Firms are supposed to do business on the market and households are not supposed to do anything but to consume and pay taxes. Some of the consequences are well-known and much discussed. Let us mention two examples of great significance.

Example 1: production of goods and services within the households, with all its economic and social implications. J.K. Galbraith says, "The common reality is that modern household involves a simple but highly important division of labour.

...the servant-wife is available, democratically, to almost the entire present male population. Were the workers so employed subject to pecuniary compensation, they would be by far the largest single category in the labour force." (Galbraith 1973, pp. 33, 35)

Example 2: the subsistence sector in developing countries. For instance, it is highly important for a country to be identified as "least developed" by the United Nations since this category is entitled to preferential treatment in various matters, e.g., in the distribution of official development aid. The major criteria applied for such identification is an upper limit of per capita GDP. On the basis of this criteria, Djibouti, for example, was several times refused to be identified as "least developed" as its per capita GDP exceeded the limit. Recently, a UN study pointed out that the limit cannot be applied to this particular case since "...the economy of Djibouti is entirely monetised which...accounts for a higher nominal per capita GDP than in countries which have large subsistence sectors insofar as in these countries income generated in the subsistence sector are not fully reflected in monetary GDP." (United Nations 1982)

To know what we are ignorant of is better than not even to know that, but it certainly does not provide the required knowledge. The feedback from National Accounts to theory should not be underestimated. One cannot analyse the unknown quantities and cannot enlighten theory without analysis. The bulk of quantitative macroeconomic research--not to mention forecasts--is based on National Accounts data in each country.

Here it is important to note that non-market activity should not be identified or mixed up with what is usually called the "second" or "black" or "underground" economy. The latter *is* market-activity, only it is tax evading and therefore, sometimes also for other reasons, illegal. Non-market economic activity on its part is performed *within* the socio-economic units, it does not enter inter-unit, social division of labour neither legally nor illegally.

On the other hand, the market is more than just the place where things are exchanged. Throughout this paper the term "market economy" is used in the broad sense, including monetary and financial superstructure, income redistribution through national and local budgets (called the "grant-economy" by K.E. Boulding); briefly speaking including everything that goes with the modern market which created modern money.

1.2 Towards A Broader Economics

Western economic literature of the past 5-10 years seems to be so much aware of and so much concerned about the unsatisfactory performance of the theory that it could be justly called the "crisis-literature". A few titles speak for themselves: "The Sad State of Orthodox Economics" (Sherman 1974), "What's Wrong With Economics?" (Gruchy et alia 1980), "The Crisis in Economic Theory" (Bell 1981).

The profession appears to be as divided along as many lines as it has ever been, but this at least gives the reader the advantage of having the critique of every school by almost every other school. It is indeed difficult to think of any aspect, any shortcoming or failure that has not been mentioned in the discussion. One even gets the discouraging impression that economists spend more time thinking about other economists than about economy.

However, an encouraging main stream is becoming evident: a common, almost general, wish to let in some fresh air, to enlarge the scope of the theory. In some cases this is just about making room for something that should have always been there trivially, for example, making room for money in economic theory (!)[6], or for the interaction between distribution and efficiency.[7] In many cases it is about returning to matters that were once there, in the classical tradition, but were forgotten or neglected for a long time, matters like the distribution of income, wealth and power in society; that is, returning to political economy from the would-be "value-free" economics. Some speak explicitly of political economy (Franklin 1980,

Jalladeau 1980, Stone 1980), but we also find "instutional economics", "instrumental economics", "interpretative theory", "social economics", "economic sociology", even "integrated social science". The trend seems to be clear although a name is yet to be found.

There are also attempts to bring in something that has never been there, to enlarge the scope of economics as such, not just the scope of this or other school. In most cases these attempts point to the same direction: to social issues beyond the political superstructure, to the human aspects of economy.

Within this stream there is even a world turned upside down: one can find serious attempts to explain love and hatred in terms of marginal utility. Naturally, most of the stream works the other way round: for example, to explain consumer behaviour in terms of human psychology is certainly a much more promising idea.[8)] Nevertheless, one need not go as far as psychology to look for territories that at present lay outside the frontiers of conventional economics which will have to be incorporated into the main body of a future, more meaningful economic theory.

History, demography, human anthropology, and sociology have much to offer. They already cover a good part of the borderline territories, they provide a vast amount of raw material for economic interpretation.[9)] On the other hand, there is an increasing number of methodological and empirical studies by economists who are determined to investigate facts and find themselves limited by the narrow concepts of conventional economics. It is not surprising that most of this type of activities is linked in one way or another to practical use of National Accounts or to building quantitative, analytical models for practical purposes.[10)]

Some economic thinking and formal model building have already started to penetrate the economic aspect of human life. The term "human capital" had already gained some respect in better times when society was busy educating more and more

young people at higher levels. Recently, with increasing number of elderly people and with permanent inflation, the social security system has become the first issue where the historically unprecedented interdependency between human life-cycle and the financial superstructure cannot be neglected any more.[11)]

Indeed, there is so much of these various promising beginnings around that one is inclined to wonder: has not the time come for a new synthesis? The right answer would probably be'no, not yet'. Before then at least two fields of outstanding significance would have to be covered systematically. One of them is the non-market economy, be it the household, the own-account production and consumption in farms, the subsistence sector in developing countries, or anything else. The other is the human life-path, more precisely, its economic implications, including the need for childrens' care, education, health services, etc. Even the broadest economic theory in the conventional sense would be open-ended at two points: at one end, human labour appears from nowhere, at the other end, human consumption disappears to nowhere. These two ends should now be conceptionally connected through the human life-path which is the source and the purpose of human economy itself.

These two fields are interrelated in many ways. Obviously, people live in some kind of socio-economic unit--family household or tribal village--that was traditionally the scene of economic activity, of production and consumption too. No matter how much of this activity has entered the inter-unit division of labour through the market, much of it has remained within the unit. Non-market economic activity, in other words, intra-unit economic activity is mostly, although not exclusively, connected with facts and needs of human life. *Vice versa*, most services required for sustaining human life, for example, the care for children, the sick and old people, are mainly provided within those units rather than through the market.

Non-market economy and "human-life economy" are also connected in a sense painful from the point of view of the much required economic analysis. For both, not only proper concepts are lacking, but also problems of valuation and measurement

are unsolved. (One should of course be reminded that permanent inflation has wiped out the role of money as a sensible, relatively stable *numeraire* even for the market-segment of the economy. Sure, there we have prices for everything, but today's prices have little to do with yesterday's prices.) So much at least seems to be plausible that the solution of the valuation-problem lies somewhere precisely in the connection between these two fields. One cannot valuate non-market performance without knowing the cost of human labour and one cannot valuate human consumption without knowing the value generated by non-market labour.

This interrelation is one of the reasons why the economic aspect of human life-path has to be incorporated into economics. Another reason is the fact that with increasing life-expectancy the human being is more and more becoming the longest lasting economic asset--and surely one of the most expensive ones.

May be the Menace of Methuselah is not so frightening as painted by K.E. Boulding. After all our own present is already an age of Methuselahs in comparison to the past. (Romeo was 16 and Julietta was 14 in the greatest love-story of all times--today they would be just high-school kids. In 1703, 0.6% of the population of New York City was recorded to be 60 years or more[12], today this ratio is around or well above 20% in most industrial countries.) But menace or no menace, a human life-span of 70-80 years or even longer, is certainly becoming the major carrier of long-term economic dynamics.

Performance provided and consumption absorbed are not distributed in a parallel manner along the life-path. If we include non-market performance, the distinction between "active" and "non-active" ages will not be that rigid as it seems to be now, but it will still remain true that in the first and in the last period of life a person absorbs more of labour (goods, services) provided by other members of society than he himself provides. In between he has to make it more than even. More than even, since if the average human being provided exactly as much as he consumed during his whole lifetime, we would still be

living before the neolithicum. Before it and not in it, since already the first piece of stone polished into a tool required human performance not consumed, required accumulation or saving, whichever side of the same process one prefers to stress. What are the proportions between those periods of life, the proportions between performance and consumption within each period, the proportions between the corresponding ratios of various classes and other social groups? How will they change and how should they change for society to be able to maintain balanced survival and progress when life expectancy increases and the age distribution of the population is in permanent transformation?

Dynamics and statics are here very much intermingled. What is the course of events on a long life-path from one point of view, is inter-generational income transfer in any given moment from the other point of view. It is also obvious that these processes must affect the secular trend of the aggregate savings ratio somehow. The direction of the effect is not so obvious. One could make a good case for an increasing ratio saying that people will have to save more since they will have to provide for a longer period of retirement. But one could make an equally good case for a decreasing ratio saying that in every given moment there will be more old people to be supported.

These are not psychological or moral or emotional problems, they are hard economic questions. It is impossible to answer them without extending economic theory onto the economic aspects of human life. Of course, we shall have to be careful. While economics will have to consider the cumulated lifetime performance and consumption by human beings, it will have to make it absolutely clear that this is not a basis in itself for social values or moral judgments. Neither a high, nor a low performance per consumption ratio in itself makes a person more valuable or more respectable to the society.

It is also clear that these and similar questions cannot be answered by picking a few phenomena and constructing "human-life economy" models, as well as non-market economy cannot be understood without its interaction with the market. They all have to

go together and before their interrelation will be understood they have to be described. This brings us to the problems of methodology.

1.3 On Methodology

Nowadays, it is fashionable to attack economic models--the small, theoretical ones for being small and abstract, the large, numerical ones for being large and empirical, both types for being irrelevant. Some kind of anti-quantitative mysticism is even becoming a sign of scholarly thinking.

It is high time to draw the lines between the concept of a model, the economic content or assumptions of particular models and the role of mathematical tools. No science can exist without models, be they formalized or not, simply because reality in its entirety cannot be perceived. (Those who speak against models think in terms of other models themselves, only they do not bother to specify their assumptions.) The actual content of a model is another matter. If some models are unrealistic or irrelevant it is not because they are models but because the particular assumptions are unrealistic and the theory concerns itself with irrelevant questions.

Finally, mathematics is a bad master but a good servant as we all know. To subordinate the subject matter to the mathematical form, to introduce impossible assumptions only in order to be able to use available techniques or to reach an elegant solution--that is really an unforgivable sin in economics. As long as one does not commit this sin, the formal presentation of a model contributes to clarity, enforces intellectual discipline, helps to specify the underlying assumptions. This is useful even if it turns out that there is no solution to the particular mathematical problem or rather that it cannot be handled with the mathematical tools available. If the problem is relevant and the formulation is precise, mathematics will have to and will be able to develop appropriate tools sooner or later.

I believe that the crucial problem of economic methodology is somewhere else. The most important thing to do would be to clearly separate assumptions from the observed or observable facts--and this is often ignored. Obviously, no theory can exist without assumptions for the very same reason as science cannot exist without models. But science and theory are not identical. Science also has to deal with facts, to observe them, to describe them, to analyse them and only then comes the theory, based on assumptions, to explain the why-s and how-s. Therefore, assumptions should be distinguished from observations.

These trivialities are not widely recognized in economics. (Perhaps this is why in most universities Economics belongs to Arts and not to Sciences.) On the contrary, being interested in facts of economic reality is often regarded as narrow-mindedness both in academic life and in economic policy-making. Only simpletons busy themselves with "accounting models". Modest attempts to enforce elementary consistency in plan-computations have been called "plan-bookkeeping" as an offence rather than a compliment. The chief economist of the OECD has her critics because "She is always digging deeper into the details and the broader implications of economic policy, whereas some governments want clearer-cut answers..."[13)]

Precisely because the general tide is such, one has to really appreciate the work of three outstanding economists who spent their lifetime in speaking, writing, lobbying for more facts, in designing and implementing the empirical evidence what we actually have: the historical time series, the Input-Output tables and the System of National Accounts. They are, of course, Simon Kuznets, Wassily Leontief and Richard Stone. They have also said everything what is worth saying of assumptions *versus* facts in economics, there is no need to repeat them further. (Leontief 1928 and 1970, Stone 1980)

The point to be made here is that for distinguishing between assumptions and observations we need a framework to describe facts and this framework must be not only consistent but also comprehensive. (I should gladly say "total" if this word did not carry some undertone to which most economists are again hostile.)

The most implicit assumptions, and usually the most crucial ones, concern not what the theory or the model is about, but what it is *not* about. Obviously, no theory or model can be about everything, but it should always be made absolutely clear: what is neglected. Also, we cannot ask for a list of the not-considered matters at the beginning of every paper, that list would be longer than the paper itself. What we need is a comprehensive descriptive framework as a commonly accepted background. In terms of this it would be easy to explain what particular segment of the economy is dealt with and what are the immediate links to other segments that are not considered in the particular theory or model. It would then not be easy but absolutely necessary to explain further what exactly has been assumed about those loose ends, the immediate links and about the impact of the neglected interdependencies upon the segment under discussion.

If such a procedure could be generally applied and to follow it would be a moral and scientific obligation as hard as, for instance, proving a theorem in mathematics, then economics as a science would greatly benefit--should we say it would graduate?

One cannot define easily such a descriptive framework. This would take time and interdisciplinary effort; by definition it would never be completely finalized since the economy is ever changing and changes would have to be reflected in a framework, that has to describe economic facts. A few basic aspects of its being comprehensive, however, can be pointed out.

A comprehensive descriptive framework should cover *the economy as a whole*, all of its parts, segments and layers. It should account for production and consumption, exchange and income transfer, money and finance; for market, non-market and "human life" economy alike.

Such a framework should be able to account for *duality* in human economy. This term is being used in various senses, the most common usage refers to the mathematical equipment. Here, I have something else in mind, something more in the classical tradition of political economy.

Economy is the mode of interaction between man and nature, on the one hand, and the mode of interaction between man and man on the other. It has its technological side and its social side. This basic duality leads to various kinds of sub-dualities, so to speak, and is reflected in various ways in economic theory and practice. The Marxian dichotomy of value and use-value, of concrete (physical) work and abstract (social) labour, for instance, is the analysis of such a sub-duality. With the advancement of the financial superstructure, however, even the social side has been split into various dual sides. For example, look at a building. As a physical object, it is the product of human work, say the work of bricklayers, plumbers, etc., and it provides shelter, for instance, it houses an assembly-line. As an economic asset, it represents accumulated value, embodied wealth and it is somebody's property. But if you look into the books of the proprietor you may find that the real owner is a bank behind him, if you look into the books of the bank you may find a third party, beyond the third party there are numerous fourth parties, and so on. The present vulnerability of the global economic system is created precisely by this endless chain, much more than by surplus of shelters or shortage of energy.

Both the shelter-asset and the asset-equity duality has to be properly treated by a comprehensive descriptive framework. The former is more difficult since there measurement and valuation, the incompatibility of various physical units of measurement, the gap between technological thinking and economic thinking create problems to which we do not yet have solutions. For the latter duality at least an ingenious device had been invented already in the Middle Ages by practical minded Italian merchants, although a self-respecting economist would seldom go anywhere near double-entry bookkeeping. With the obvious result that the two sides of a dual phenomena almost always get mixed up, without even realizing the confusion. (One of the marvel-pieces: the gross domestic *product* is defined as the sum of *incomes*.)

A comprehensive descriptive framework should place economic activities *in time*. It should have room for both stocks and flows, "statics" and "dynamics". It should be able to describe situations at a given point of time, course of events over a period of time, complete life-cycles and intertemporal interdependencies among various structural segments. This requirement is so obvious, it does not need to be further elaborated. To implement it will be another matter.

Finally a comprehensive framework should describe the economy *as a system*. It should not just list various sets of agents and events, it should be able to show the interaction and interdependency among them. Also, a simple requirement in principle and a tremendous task for implementation.[14)]

Moreover, we are interested in not only the direct, but also in the indirect, endless circular interdependencies. These, however, cannot be directly observed and, therefore, cannot be recorded by a strictly descriptive framework; to establish them we have to make certain assumptions. At this point the framework ceases to be strictly descriptive and tends to become a model, or any number of models depending on the number of sets of assumptions we apply to it.

We do not immediately need to introduce ill-defined theoretical notions or jump into far-fetched assumptions on the past motivation and future behaviour of the elements (parts) of the system. We may still remain mainly with the facts and apply as few and as plausible assumptions as it is absolutely necessary for being able to penetrate into the endless chain of indirect interdependencies. A model built along these lines could be called an *analytical* model, or simply an analytical tool, as distinguished from both theoretical and predictive (forecasting) models.

In the next chapter I shall try to outline the crudest scheme of a descriptive framework that would satisfy some of the above requirements. It would cover all segments of economic activity, it has room for the asset-equity duality, it places economy in time, although in a rather simplistic way, and it

describes the direct interdependencies. It does not, and this is a major shortcoming, handle the shelter-asset duality because it has to assume a common unit of measurement for all segments. The link with technology and through it with nature is still missing.

In the third chapter I shall present a simple but powerful analytical tool that can be applied to the given descriptive framework. It is built on long, practical experience with input-output analysis, at first using it to the original, traditional form of the model, later applying it to quite different fields, for example, money flows, and finally generalizing the technique into a *symmetric* handling of input coefficients and output coefficients, i.e., backward proceeding and forward proceeding chains of links.[15] It should be noted that despite the conventional language, for instance, the use of the term "equation", there are no *unknown* quantities to be found in Chapter 3. What are conventionally the "variables", are here supposed to be observed facts, well-known quantities available in the descriptive framework. The purpose of the exercise is not to find a solution for the variables but to define the matrices whose entries represent the total (direct plus indirect) effects of each variable on every other variable.

2. THE DESCRIPTIVE FRAMEWORK

2.1 Basic Concepts

A STOCK is the value of the existing amount of a given economic substance at a given instant of time. Time is not considered continuous, it is measured in distant intervals. For the sake of simplicity, we might happen to speak of a year, but in principle the time period can be shorter (a quarter year, a month, etc.) or longer (any number of years, a decade, a century, etc.). For each time interval t and for each substance i we define the OPENING STOCK to be denoted by $s_i(t)$ and the CLOSING STOCK to be denoted by $z_i(t)$.

A practical working model will have to account for discrepancies between the closing stock of a period and the opening stock of the next period, but here in this brief outline we assume

$$s_i(t) = z_i(t - 1) \tag{1}$$

For brevity, time indices will be omitted in the following for as long as they will not be explicitly needed to describe intertemporal relations.

Following the argument in Chapter 1 on *duality* we distinguish between stocks of ASSETS and stocks of EQUITIES, the terms borrowed from double-entry bookkeeping. It should be kept in mind, however, that an economic object is not *either* an asset *or* an equity: it is part of an asset from one point of view *and* part of some equity from another point of view. This is the essence of duality. Therefore, if s_{ai}, respectively z_{ai}, denotes the stock of the j-th kind of equity, then for every economic unit or system

$$\sum_i s_{ai} = \sum_j s_{ej} \tag{2a}$$

$$\sum_i z_{ai} = \sum_j z_{ej} \tag{2b}$$

that is, *the total value of assets is equal to the total sum of equities.* This may be regarded as the basic equation of duality.

The asset-aspect will tell us *what* the economic objects are, what *physical form* they exist in if they are TANGIBLE assets, such as fixed capital (buildings, structures, machines, etc.), and inventories (raw materials, finished goods, unfinished goods in production, etc.) or what specific *legal and income-providing characteristics* they have if they belong to the mysterious world of FINANCIAL assets, such as deposits, bills, bonds, shares, etc.

The equity-aspect gives a different breakdown of the total value of all assets. It tells about the *source* of that value in terms of property, of claiming rights. A part, or the whole, of the total value may be unconditionally *owned* by the given economic unit--that part will be called PROPERTY. Another part or the whole may be *lended* by outside creditors under various conditions--that will be called FINANCIAL LIABILITIES.[16)]

In addition to these conventional notions, HUMAN assets and equities will be introduced in Section 2.3. Within the major groups that are denoted by block letters, a long list of specific kinds of assets and equities would have to be defined for a model to become really operational. For a rough outline to be presented in this paper, only a few very aggregate subgroups have been defined so that meaningful examples could be described. The classification is given in Tables 1a and 1b.

A FLOW is an event or transaction which affects two stocks simultaneously. With respect to distinct time intervals, a flow is the cumulated value of those events or transactions which affect the same stocks in the same way during the time period.

For most economists the term flow will instinctively convey the notion of *transition:* a flow of products, or incomes, or funds is coming *from* somewhere and goes *to* some other place, that is, one stock is decreased and another is increased. The physicist or the mathematician will immediately think in terms of a transition matrix: some particle flows--transits--from one state to another. The accountant, however, knows better, because he is familiar with the double-entry duality. He is familiar with transactions that increase an asset *and* an equity simultaneously while nothing gets decreased, or the other round, transactions that decrease an asset *and* an equity at the same time while nothing increases. As if something has flown out of nowhere or something has vanished into nowhere.

There is nothing mysterious about this. On the contrary, *dual-increase* flows represent, among others, the very notion of economic growth. Just consider: if the increase of one asset would necessarily require the decrease of another asset, the total value of assets would never grow, the economy would never expand. If the decrease of an asset would necessarily imply the increase of another one, things would have to be ever-lasting. *Dual-decrease* flows represent, among others, losses and damages due to fire, earth-quake or human negligence. (If a house burns down, both asset and equity perish.) In other words, dual-increase and dual-decrease flows are not bookkeeping technicalities, they are facts of life. Without them the dynamic process of economic growth cannot be described properly since transitions alone can only alter the structure of a system but cannot make it expand or contract.

Double-entry bookkeeping does not create dual-increase and dual decrease flows, it just provides an ingenious device to deal with them as well as with the transitory flows in a simple unified framework. (It is also ingenious for its instinctly apologetic service to the capitalist economy: by recording profit as a dual-increase flow within the capitalist firm it helps to hide the true origin of capital and wealth. This is,

however, no reason to refuse to apply the clever device if there is a theory to provide proper interpretation.) The device is as follows.

We reverse the relation of assets and equities to increase and decrease. Thus, we only need two kinds of "entries" for describing four types of flows and their impact on stocks:

DEBIT (ENTRY) means increase in case of an asset and decrease in case of an equity,

CREDIT (ENTRY) means decrease in case of an asset and increase in case of an equity.

Let us see how it works:

		The Stock To Be Given	
		Credit	Debit
Transitory flow (assets)	ρ	asset ↓	asset ↑
Transitory flow (equities)	π	equity ↑	equity ↓
Dual-increase	α	asset ↑	equity ↑
Dual-decrease	ω	asset ↓	equity ↓

Of course, the upward pointing arrows mean increase, the downward pointing ones stand for decrease. The Greek letters are symbols to be used in the following for brevity. The message of alpha and omega is, I hope, obvious: α stands for birth or origin and ω for death or end.

If we denote by f_{ij} a flow for which a credit entry has to be made in the record (account) of stock i and a debit entry in the record of stock j then

$$s_{ai} + \sum_j f_{ji} - \sum_j f_{ij} = z_{ai} \tag{3a}$$

$$s_{ej} + \sum_i f_{ji} - \sum_i f_{ij} = z_{ej} \tag{3b}$$

That is, *opening stocks plus incoming flows minus outgoing flows equal closing stocks.* These may be regarded as the basic equation of the stock -flow relationship.

We shall also make use of a clever detail in bookkeeping, namely, we shall assign records to stocks that do not exist. These records, to be called NO-STOCK assets or equities, can be used for cumulating, reallocating flows. Although each of them is supposed to be balanced by (or, at least at) the end of the time-period since in principle the corresponding kind of stock is non-existing, at any given point of time during the period these records may be unbalanced, therefore, without them equation (2) would not be satisfied.

We shall not follow, of course, the tiresome procedure of keeping separate records for each stock and thereby having to account twice for each flow, a procedure applied in bookkeeping still today. (An amazing anachronism!) We shall apply the more convenient matrix-notation. The particular structure of the matrices and vectors will, however, depend on what exactly we are talking about. Designing them is, therefore, left to the following sections.

It is important to note that so far *no assumptions have been made* and in this sense *no model has been built.* We derived some key concepts from the reality of everyday economic life. We selected a formalism convenient for recording observable facts and in equations (2) and (3) formulated some trivial accounting identities.

The following sections of this Chapter will attempt to show how these tools can be used for describing and interpreting various economic processes at various levels.

2.2 The Unit Of The Conventional Economy

The unit to be described here belongs to the conventional economy in the sense that it is limited to conventional economic activities such as defined in double-entry bookkeeping or in the System of National Accounts. Our unit is, however, not quite conventional in the sense that it is not limited to some kinds of specialized activities. It may produce missiles or hamburgers, it may trade, lend and borrow money, it may employ outside labour and/or sell labour to outsiders, it may consume, invest and save, it may receive or supply or intermediate income transfers. Of course, we need not assume that all units really engage in all these activities, we just do not assume the opposite.

There is only one single activity that is prohibited to our unit: it cannot issue money. This is reserved for a special unit to be called the Bank--its activities will be described later.

The general-purpose unit in this section is separated from other units by ownership but is linked to them by some degree of division of labour. It may engage in all kinds of activities, but it is not an autarch, isolated unit. In other words, it has market or money relations with the outside world (although its external links are not limited to the market), but *there is no market within the unit.* Allocation of resources within the unit will not be mediated by money, it will follow the decisions made by the management or by the housewife or by some decision-making body. (Please note that state and local budgets also come under this definition--they are included in the notion of a unit.)

With some lack of preciseness, it may be said that the motivation behind those decisions is the subject of microeconomic theory. Here, however, we are not concerned with the motivation of behaviour, we are concerned with describing the activities.

The flows recorded within the unit will be denoted by u_{ij} (credit to stock i, debit to stock j). We organize the matrix notation by defining the flow-matrix:

$$U = \{u_{ij}\} \qquad i = 1,\ldots,N_s$$

$$j = 1,\ldots,N_s$$

The stock vectors:

$$\hat{s}_a = \{s_{ah}\} \qquad \hat{z}_a = \{z_{ah}\} \qquad h = 1,\ldots,N_a$$

$$\hat{s}_e = \{s_{ek}\} \qquad \hat{z}_e = \{z_{ek}\} \qquad k = 1,\ldots,N_e$$

$$s_a = \begin{bmatrix} \hat{s}_a \\ \underline{0} \end{bmatrix} \qquad z_a = \begin{bmatrix} \hat{z}_a \\ \underline{0} \end{bmatrix} \qquad \underline{0} \text{ zero vector of order } N_e$$

$$s_e = \begin{bmatrix} \underline{0} \\ \hat{s}_e \end{bmatrix} \qquad z_e = \begin{bmatrix} \underline{0} \\ \hat{z}_e \end{bmatrix} \qquad \underline{0} \text{ zero vector of order } N_a$$

where

N_a number of asset stocks

N_e number of equity stocks

$N_s = N_a + N_e$

in addition we need

$1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ and ' for denoting transposition.

With these notations we have in place of (3a) and (3b) the matrix form of the basic stock-flow relation for the unit:

$$s_a' + z_e' + 1'U = (s_e + z_a + U1)' \qquad (4)$$

Figure 1 has been designed to give the reader a visual impression of this descriptive framework. What is important to note is the partitioned structure of the flow-matrix: blocks U_ρ, U_α, U_ω and U_π correspond to the four types of flows discussed in the previous section, they serve as the logical "place" for the respective flows.

The flow matrix may be regarded as a very much generalized and somehow twisted input-output matrix. For assets, the entries in their columns represent inflows to the stock and entries in their rows represent outflows from the stock. For equities it is the other way round: row entries stand for inflows and column entries stand for outflows. It is exactly this "twisted" pattern that distinguishes our flow matrix from the usual transition matrices. Although blocks U_ρ and U_π may be regarded as open transition matrices: open in the sense that they receive external inputs, respectively unload external outputs, to blocks U_α and U_ω.

Of course, to get a better understanding, or at least feeling, of these implications one would have to go into details. For this purpose a tentative, illustrative classification of stocks (Table 1/a) and a list of selected flows representing various economic activities (Table 2/a) has been prepared. Figure 2 depicts the corresponding flow matrix. Stocks are identified by three-digit symbols, flows by two digits. Here we only call attention to a few interesting examples:

Flows 11 and 12. The simple classical market transactions. Also, a classic example of transitory flows: the form of the accumulated wealth changes from money into goods (11), respectively, from goods into money (12). Equities remain intact. Obviously, 11 and 12 appear in the U_ρ quadrant of the flow matrix.

Flows 34 and 53. The two ends of the labour market (cash payment for simplicity). Buying labour seems not to be different from buying goods: money is changed into the value of the products produced with the help of the employed labour (flow 34 in block U_ρ). Selling labour, however, is different: new assets (money)

and new equities (current income) have been created. Flow 53 appears in block U_α.

Flows 83 and 84. The simplest examples of income transfer. Receiving income (83) appears in block U_α while giving it away--losing both asset and equity--shows up in U_ω.

Flows 31 to 36. The costs of production are accumulating in the inventory of own-produced goods. Column AT3 may even be regarded as the data bank for the conventional production function of the unit.

Flows 41 to 43. They show how the "classical" interpretation of the market transactions is false. For them to be so simple, with flow 12 we had to assume the "no-profit" case. Very convenient for theory. But in real life who would sell for no profits? The inventory of goods produced in the unit is valuated at costs in every decent firm. Therefore, when selling the product, the inventory can be decreased only with the cost of the product while the stock of "accounts receivable" (future money to be received from the customer) will increase with the selling price. The balance (flow 43) has to be accounted for, and, naturally, it appears in U_α: the profit is born.

Flows 61 to 65. There are two points to be made here. First, this is an example of the no-stock record: we cumulate everything consumed in AN1 as if such asset existed. This may provide the data for a conventional consumption function and it simplifies matters at the end of the period when we have to write down consumption against (as an outlay of) current income. The second point: in conventional economy consumption is a loss. (Flow 65 appears U_ω.)

It would be worthwhile for the truly interested reader to find out the difference between various sub-blocks of the main blocks. The intersection of real assets with financial assets, for example, carries economic meaning quite different from that of the main diagonal blocks, and so on. The general reader, however, should not be bored with more details.

2.3 The Unit Of The Human Economy

It is of course not a separate unit. It is the very same unit as that of the conventional economy except that it

- also engages in activities not permitted by National Accounts, such as preparing meals, doing the dishes, repairing the vacuum-cleaner, caring for the children, for the sick and elderly members of the unit, etc.;
- does not consider human consumption a loss ("final" output or unproductive outlay on the current income account) but keeps records on how consumption is accumulated into human assets;
- does not regard income and property as coming from nowhere, but traces them back to their origin, that is, to human performances.

We define HUMAN ASSETS as the value of cumulated lifetime consumption of human beings alive and HUMAN EQUITIES as the value of cumulated lifetime labour performed by human beings alive. In this section we only have to show how these concepts can be fitted into and accounted for within the descriptive framework developed for the economic unit.

The further breakdown of human assets and equities can be left for actual model building depending on the purpose of the model. The most detailed accounting would be to pin the stocks to individual persons--perfectly all right in principle, but obviously impracticable if more than one particular household is to be described. We may think of aggregation by age-groups or by intervals of the human life-cycle. This solution would be particularly attractive because it would provide a direct link to the underlying demographic observations. Transitions on the life path would appear as transitory flows in the respective main-diagonal blocks of the flow matrix--only they would be measured in terms of assets and equities carried over to the next life-interval rather than in terms of the number of persons making the transition.

One distinction, however, has to be made even at the present tentative level of outlining a descriptive framework. Namely,

for one unit we must distinguish between the stocks that belong to persons resident in the unit and the stocks that had been carried over from or to other units by migration. The latter type will be called CONTRIBUTIONS rendered or received. This notion is necessary because the value of the asset a person carries would only exceptionally equal the value of the equity carried by him or her.

Suppose, for example, that a person leaves the unit at an early age obviously carrying more assets than equities. If for the sake of simplicity we assume that conventional assets and equities of the unit balance, then this would imply that the remaining resident members of the unit carry more equities than assets. (For example, the grown-up, but still young, child leaves the family carrying assets embodied in his or her person that originated in the parents' performance.) Now the unit can write down--in the U_{ω} block of its flow matrix--as much assets as equities. But the surplus performance of the remaining members cannot be written down since those persons are still alive and resident in the unit. The surplus assets carried away by the person leaving the unit must be somehow kept on the books although they cannot be regarded as assets embodied in residents any more. This surplus would then be regarded as "contributions rendered", namely, contribution to human assets embodied in persons resident elsewhere, outside the unit.

If the interested reader will now care to think over both the opposite case (of surplus performance carried away) and the situation in the opposite unit, he or she will see that the records are kept straight everywhere.

Having accounted for migration we still have to deal briefly with the two major yardsticks of human life, birth and death.

Surprisingly, but logically, the birth of a human baby does not show up in the U_{α} block of our flow matrix. If we start recording the assets embodied in a person, the costs (hospital, etc.) of birth will appear in block U_{ρ} as well as any later ordinary consumption by the person. This is all right and logical since we do not identify the human person with consumption

and performance, we do not account for human life in the biological, psychological, cultural or any other sense, but in the economic sense. When a baby is born, a human being is born, but the accumulation of economic objects, assets and equities related to the person will have to come later.

Death is different. At the time of death, a person will already have accumulated at least some consumption, but most probably some performance too, and this has to be accounted for. The case is similar to that of emmigration, only the remaining balance between assets and equities that are carried to the grave cannot be considered human stocks anymore. This remaining balance has to be written down from, or added to, the conventional property of the unit. Again, a systematic consideration of all records involved would prove that things check out properly.

To return to the formal aspects of our descriptive framework, the notation and equations introduced in the previous section need not be changed. All we have to do is complete the classification of stocks (given in Table 1/b) and provide a few selected examples of human flows (given in Table 2/b) as well as an expanded flow-matrix (Figure 3) to help with the homework of the interested reader.

2.4 The Bank

The framework developed so far for the general purpose unit of the human economy is ready to describe each and every economic fact and process except one: the birth and death of modern money which is not the good old gold anymore.

This is the reason why the column and the row of the equity termed "Money issued" (noted by EF1 on the Tables and Figures) so far remained empty in the flow-matrix. An ordinary economic unit cannot have money as equity, it can only hold money as an asset.

However, in every country there is at least one central monetary authority that has the legal right to create money and

for this particular--and special--economic unit money serves as equity. We shall call this unit the Bank. (Even an international money-making machinery called the Special Drawing Rights has been created. It does not serve the purpose it was set up for very efficiently, but it does exist. Of course, we cannot go into such details here.)

The creation and the withdrawal of money is a very simple act and it is not difficult to account for it in our descriptive framework. (To understand the implications is another matter.)

Suppose that some other unit, to be called the Debtor here, comes to the Bank and applies for a loan. The Bank approves a transferable deposit on his account. The money has been created. The Debtor takes it and spends it. When he later comes in the possession of money again (because his investment turned out to be profitable or because he has some other source of current income) he transfers the money to his account at the Bank and says he is ready to pay back the loan. The Bank accepts it and he is no longer in debt. The money has vanished.

The Bank will make the following entries in its records:

	Debit	Credit
When issuing	Loan to Debtor (AF3)	Deposit of Debtor (EF1)
When withdrawing	Deposit of Debtor (EF1)	Loan to Debtor (AF3)

The Debtor on his part will make the following entries in his flow-matrix:

	Debit	Credit
When getting the loan	Money (AF1)	Loan from Bank (EF3)
When paying back	Loan from Bank (EF3)	Money (AF1)

Naturally, the above story is an oversimplification. A workable model would have to account for all the intricate devices, flows and stocks of modern finance. Here we only had to show that our descriptive framework is able to cover this very important aspect of economic reality too.

I would like to point out that the money issued--that is, the amount of money in circulation--serves as an equity for the Bank, which again is no bookkeeping technicality. Since money is not gold or any other object of real value but just a claim against the Bank, all those who hold money as asset are in fact lending it, out of their own equities, to the Bank, although the Bank is only an intermediary: it has lent money to the Debtors and its loan serves as equity for the Debtors. The circuit is closed: those who keep money on their accounts or in their pockets are actually financing those projects that are apparently financed by the Bank--investments or government deficits. In other words, the creation of money really creates equities somewhere for somebody. It certainly cannot create real wealth. As a counterpart of those additional equities, it also creates financial assets, the money itself. But in doing so, it redistributes real income and real wealth within the economy.

2.5 Macroeconomic Aggregates

Surprisingly, there is little to say on this.

We disregard the practical difficulties faced by statisticians. They have a hard job putting together national accounts as economic units actually fall into various categories with regard to the records (if any) they keep, and the data (if any) they are required or requested to report.

However, if in principle we simply sum up the flow-matrices and stock vectors of all the units as described in the previous sections, we get the national aggregates. If the classification of stocks, and particularly of no-stock records would be conveniently designed even the major accounts, notions and categories of the present SNA system would directly appear in

their required form, but we would have much more. The kind of flow-matrix as shown in Figure 3 describing a whole national economy rather than a single unit would certainly provide a concise, organised, meaningful description of macroeconomic flows.

Naturally, at the macro level things get a new dimension, they acquire a new significance. This is trivial with regard to the conventional economy. The conceptual difference between GDP and the net earnings of a farmer, or the difference between the notion of aggregate personal consumption and Mrs. Smith's weekly shopping at the supermarket would be obvious for every economist. The same is true for the newly introduced human stocks and flows. Just think of the gain in human "contributions received" for an economy that has a constant inflow of grown-up, trained and educated people at the prime of their working life who carry more human assets than equities with them and who will reverse their balance in their new home country. Or, think of the amount of equities frozen forever into assets "contributions rendered" inthe countries that have a constant outflow of people.

It is of course not necessary to reach the national level in one big jump. One could think of all kinds of intermediate, sectoral, regional aggregates. If, for example, units would be grouped according to the presently prescribed classification of transactors, it would be interesting to see how much of the economic activities actually carried out is neglected under the present system simply because various kinds of units are not supposed to do this or that.

Aggregation, however, does not build the bridge between micro and macro. Precisely because the aggregate categories have a new, specific quality, the fact that the numbers can be traced back to individual units does not in itself explain how this new quality has come about, what it means, how the economy works, what the relation is among the various individual units.

2.6 The Economic System

The economic system is a set of interactive and, therefore, interdependent units. In this section we attempt to extend the descriptive framework to the system of interdependencies.

First, we have to account for the fact that now we deal with more than one unit. For this purpose every symbol used to describe one unit in Section 2.2 has to be further denoted by an upper right index indicating the serial number of the unit so that s_a^n, s_e^n, z_a^n, z_e^n will stand for the stock vectors and U^n for the flow-matrix of the n-th unit. We get rid of this upper right indices immediately by defining

$$s_a = \begin{bmatrix} s_a^1 \\ s_a^2 \\ \cdot \\ \cdot \\ s_a^{Nu} \end{bmatrix} \qquad s_e = \begin{bmatrix} s_e^1 \\ s_e^2 \\ \cdot \\ \cdot \\ s_e^{Nu} \end{bmatrix}$$

$$z_a = \begin{bmatrix} z_a^1 \\ z_a^2 \\ \cdot \\ \cdot \\ z_a^{Nu} \end{bmatrix} \qquad z_e = \begin{bmatrix} z_e^1 \\ z_e^2 \\ \cdot \\ \cdot \\ z_e^{Nu} \end{bmatrix}$$

and

$$U = \begin{bmatrix} U^1 & & & \\ & U^2 & & \\ & & \ddots & \\ & & & U^{Nu} \end{bmatrix}$$

Nu the number of units

Figure 4 will help to realize that equation (4) in Section 2.2 holds for this enlarged system. (Rather than changing the notation used in (4) we changed the definition of the individual symbols.) They stand not for individual units anymore, but for the whole economy as an ordered set of the units.

Second, we have to make another important distinction between two types of flows. Each flow recorded by individual units is either a *connecting flow* linking the unit with one or more outside parties (other units) or a *domestic* flow, an internal transaction within the recording unit. Obviously, any market transaction--selling or buying on commercial, labour or financial markets--will be defined as a connecting flow. This will also be the case with flows representing income transfer in cash or in kind. A good rule of the thumb would be that whenever it is possible to name an outside party *directly* linked to a flow then that flow is a connecting one--the rest are domestic flows.

We introduce new symbols:

$$c^n_{ij} = u^n_{ij} \quad \textit{if } u^n_{ij} \text{ is a connecting flow and}$$

$$d^n_{ij} = u^n_{ij} \quad \textit{if } u^n_{ij} \text{ is a domestic flow}$$

$$C^n = \{c^n_{ij}\} \qquad D^n = \{d^n_{ij}\}$$

$$C = \begin{bmatrix} C^1 & & \\ & C^2 & \\ & & \ddots \\ & & & C^{Nu} \end{bmatrix} \qquad D = \begin{bmatrix} D^1 & & \\ & D^2 & \\ & & \ddots \\ & & & D^{Nu} \end{bmatrix}$$

and thus we have

$$U = C + D \tag{5}$$

Third and the most important, we shall replace matrix C by some other matrix. To do this we have to consider the following.

If c_{ij}^{n} is a flow that connects unit n with unit m, and therefore we further denote it by $c_{ij}^{n(m)}$, then there must be a flow $c_{hk}^{m(n)}$ recorded in unit m such that $c_{ij}^{n(m)} = c_{hk}^{m(n)}$. (In reality, one or both of the two parties might keep aggregate records, i.e., not naming their individual partners. That would complicate our narrative but would not affect the conclusions. So for the sake of simplicity we neglect this case now.) Therefore, one transaction performed between units n and m is represented by two entries in matrix C. Their respective coordinates are n(i), n(j) for $c_{ij}^{n(m)}$ and m(h), m(k) for $c_{hk}^{m(n)}$ where n(i) denotes the i-th row within the n-th block of rows and so on. However, these four coordinates uniquely determine two other entries in two off-diagonal blocks of matrix C. Let them be denoted by

$1_{ik}^{n,m}$ with coordinates n(i), m(k)

$1_{h,j}^{m,n}$ with coordinates m(h), n(j)

and their meaning is basically explained precisely by the coordinates. (Figure 5 will help to see the pattern.) Still let us put it into words.

A flow denoted by $1_{ik}^{n,m}$ represents a transition between unit n and unit m such that it involves a credit entry for stock i in unit n and a debit entry for stock k in unit m at the same time. (An actual model would have to account for discrepancies in time, e.g., the physical time required for the movement of goods, money, people. Here we forget about this.) It is easy to see that it is correct

	Debit unit	Debit stock	Debit unit	Debit stock
to replace flows:				
$c_{ij}^{n(m)}$	n	i	n	j
$c_{hk}^{m(n)}$	m	h	m	k
by flows:				
$l_{ik}^{n,m}$	n	i	m	k
$l_{h,j}^{m,n}$	m	h	n	j

Now we define

$$L^{n,m} = \{l_{i,j}^{n,m}\} \qquad L^{n,n} = \underline{0}$$

$$L = [L^{n,m}]$$

and it can be seen that

$$1'L = 1'C \quad \text{and} \quad L1 = C1 \tag{6}$$

that is, the row and column sums of L equal the row, respectively column sums of C. Therefore, if we further define

$$F = D + L \tag{7}$$

(to be seen in Figure 6), that is, we substitute matrix C with matrix L, then the old equation (4) will hold for the new matrix F too:

$$s_a' + z_e' + 1'F = (s_e + z_a + F1)' \tag{8}$$

and with this the formal tricks have been completed. (The order of matrix F, to be denoted by N_F in the following, will be $N_F = \sum_u N_s^u$ (u = 1 , Nu) where N_s^u is the number of stocks in the u-th unit.)

What have we done in terms of economics? We have introduced a new type of flow or rather, more precisely, a new way of recording a flow. We have replaced the isolated, self-centered unit-approach to inter-unit flows by describing *directly* the linking flows that interconnect units with each other. However, we did this in such a way that the integrity of the unit has been preserved. The column of the large matrix F that corresponds to the j-th stock of the n-th unit still records the inflows to that stock if it is an asset and the outflows from it if it is an equity. *Mutatis mutandis* the same can be said for the rows. Therefore, the basic equation describing the stock-flow relation has also been preserved and this is particularly important.

The difference from the point of view of the unit is that previously a flow was recorded at one point, namely, at the intersection of the two stocks it had affected. Now it is recorded at two points indicating the two stocks *and* those two stocks that are affected in the unit which is the outside party. (For $c_{ij}^{n(m)}$ we now have $l_{ik}^{n,m}$ and $l_{hj}^{m,n}$.) If for just a minute one would be tempted to take this whole thing seriously one could say that any business firm would be grateful for this additional information.

The point is, however, not the unit but the economy as a whole. While preserving the integrity of the unit and keeping its records straight we also managed to describe the system of interdependence among units. In the main diagonal blocks the flow-matrix F describes what is going on *within* units and in the off-diagonal blocks what is going on *between* the units. Maybe, this kind of "double-entry bookkeeping at the macro level" could once provide us with the missing link between the macro and micro approaches to economy.

It is important to observe that the off-diagonal blocks of F have the same $[^{\rho}_{\alpha}\ ^{\omega}_{\pi}]$ structure which is characteristic of the original U^n flow-matrices of the units. But, here the exact pattern would be determined from two sides, namely, by the stock structure of the two interconnected units. (See Figure 7) Therefore, and this is particularly important, the $l^{n,m}_{ik}$ flows will not necessaryily appear in the same Greek letter sub-block where the original $c^{n(m)}_{ij}$ and/or $c^{m(n)}_{hk}$ appeared. The L link-matrix may reveal simple transitory flows where the participants have recorded dual-increase, respectively dual-decrease flows, that is, gains, respectively losses--it views things from the point of view of the whole economy.

At this point again a few examples might help. We only take up four of those that had been discussed in Sections 2.2 and 2.3. Each of them is supported by a figure where the original C components are shown with their original symbol while the new L components are denoted by one digit.

Figure 8a. Rather than seeing each unit keeping their wealth in a changed physical form, we now see goods flowing from unit n to unit m while money makes the opposite trip.

Figure 8b. The differing views of the two parties on the labour market appear now as two aspects of the economy as a whole. Remember: buying labour was a transitory flow for the employer (34), selling labour was a double-increase flow for the worker (53). In the L matrix we now see one transitory flow as money passes from the employer to the worker (L^{nm}_{ρ}), and we see one double-increase flow in $L^{m,n}_{\alpha}$ as labour performed by members of unit m increases their equities and also increases the value of asset AT3 of unit n.

Figure 8c. Typical example of how income transfer, gain to one party and loss to the other party, is simple transition for the economy as a whole on both the asset and the equity sides.

Figure 8d. Illustrates the case of human migration that had been discussed in Section 2.3.

It is easy to see from these examples that our L link-matrix carries various sets of information that have been used in macroeconomic--i.e., intersectoral models. Selected parts of the L matrix would more or less correspond to various models, such as the input-output table (on a commodity-enterprise basis), the flow of funds, the model of money flows, various income distribution models, etc. Here, however, we have the total picture, the ordered complexity and not only some voluntarily picked isolated part of the interdependent system.

Also, it is not difficult to see how this type of descriptive framework could serve to describe both global and national economies. The flow of goods, money and people across national frontiers is at least partly controlled by different rules and motivated by different motivations than they are within a country. However, rules and motivations are not our concern here, we attempt to describe facts. The total set of economic units described by our F matrix can be easily seen as containing units of various, or all, countries; the proper blocks of the L term of F would then record inter-country flows such as exports and imports. In this case we must deal with several Banks issuing several kinds of money, i.e., national currencies and we would have to account for the exchange of currencies--a technical detail which can be taken care of.

The total, in principle even global, complexity is not meant to actually replace specialized models that address special economic issues. One cannot deal with everything at the same time as that would only mean dealing with nothing. The total complexity is meant to serve as a framework for reference, so that every model could be clearly defined in terms of what it covers and of what it consciously, purposefully and explicitly neglects.

2.7 Dynamic Transformation

Flows represent events, transactions that occur at a given instant of time (even if we account for them by cumulating over a period of time). Flows do not describe *transformation that takes time.*

On the other hand, stocks get accumulated because things spend time in a given state. For example, we have stocks of fixed assets because buildings and equipment last for several years. In this sense, *stocks represent time.* When looking at them at a given moment, we see them as the accumulated amount of some particles. But, we may as well look exactly at the particles that have once entered the stock and will leave it later on; the accumulated amount at any given minute will depend on the time spent in the stock by the transiting particles. By particles one does not necessarily mean some integer, tangible objects. The term might, and for most stocks should be, interpreted in an imaginary way.

Now, if the particular state represented by the stock happens to be a "state" in which things get transformed, that is, apart from aging they get changed with respect to substance, quality, appearance, purpose or anything, then the stock corresponds to a transformation process that goes on in time, that is, to a *dynamic transformation.* The most obvious, and therefore the most helpful example would be the stock of *unfinished products* that actually houses the *process of production:* material, labour, and other inputs go in and the finished product comes out.

While in the previous sections we regarded the stocks as some kind of black boxes that receive inflows and unload outflows, now we look into their inside in order to get to the transformation processes. We shall have to discover the particular combination of specific inflows that produces a particular outflow and measure the time this process takes.

When attempting to describe transformations, an actual, workable model of the economy would have to face a number of problems that are conventionally not dealt with by economics.

When attempting to describe the economic aspects of transformation, one cannot disregard the very nature of that process: the technology that transforms material, equipment and labour into products; the legal implications that set the rules for when and how to spend what funds on what purposes; the biological, social and cultural factors of human life that affect the transformation in human stocks, and so on.

Here, it must be noted that records on transformation will not always represent hard facts as records on stocks and flows do. First, transformation is more difficult to observe precisely because it goes on *within* the stocks. Second and more important, the specific nature of the non-economic aspect of transformation is not always well-known, completely understood or precisely quantified. Therefore, the quantification of the economic aspect will often have to rely on estimation, sometimes guesstimate. Extreme disaggregation would reduce the need for estimation but cannot eliminate it completely, not even in principle. Example: twin-products where the cost of each of the twins can only be established by imputation. However, this is no reason for refraining from recording the estimated quantities or from providing estimates where they are missing. The significance of such estimations or imputations should not be underestimated: in reality they have a tendency to turn into hard facts. Example: the price of products will be based on costs that are partly observed, partly imputed.

Returning to the more technical aspects of our descriptive framework, we should note that while a flow is defined as a link between stocks, transformation will have to be described as a set of links among flows, namely, the link between the inflows to and the outflows from a particular stock. Since these flows are identified by the stock in question *and* the other stock that was affected by the flow, we can trace the interdependence between various transformation processes.

We introduce

$$g^{i}_{nh}(t\,,\,\tau) \qquad\qquad i,n,h = 1,\ldots,N_F$$

denoting the imaginary particle, the overlapping part of an inflow and an outflow, namely, the common part of the flows

a) Credit n, Debit i in period t
b) Credit i, Debit h in period τ

Particle $g^i_{nh}(t, \tau)$ is clearly transiting through stock i, entering with one of the above flows and leaving with the other one. If i is an asset then flow a) is the inflow and b) is the outflow, hence $\tau \geq t$ is required. If i is an equity then b) is the inflow and a) is the outflow. Therefore, $\tau \leq t$ is true.

Figure 9 may help visualize the idea and Table 3 shows how the already familiar notions can be expressed in terms of the g particles. But we also need some new concepts.

If we wish to study a particular time period t in relation to its past and future, then it is necessary to distinguish that part of the opening stock which gets mobilized in period t, that is, leaves the stock by becoming part of an outflow from the other part of the opening stock which survives period t within the stock, thus becoming part of the closing stock. Clearly, the other part of the closing stock will have to be made up of those particles that enter the stock during period t and leave it in some later period. Let

$$m^i_j(t)$$

denote the *mobilized part of the opening stock* i that leaves in t through "gate" j in the sense that the pair of stocks affected by its exit would be i and j. Further, let

$$r^i_j(t)$$

denote that part of the inflow entering i through gate j in period t which *remains settled down in the closing stock* at the end of this period. Table 3 has the proper definitions for m-s and r-s too.

So far we discussed one particular stock that may have been either an asset or an equity. As we will now return to the whole interdependent system of assets and equities, we have to consider an important fact. While in the stock-flow relations of any time period assets and equities live together as dual counterparts, their internal transformation processes over time are separated from each other. When a brand-new house burns down, an asset and equity perish (recorded in the ω blocks of the F-flow matrix); but the asset was only a few weeks old and the lost equity capital may have been inherited from ancestors. Therefore, transformation within assets and transformation within equities can be interlinked only with respect to one particular time period t, and the particles that share a common fate in t come and go on their separate ways with respect to the past and the future. This will already be reflected in notation when we turn to matrices:

$$G^i(t\,,\,t) = \{g^i_{nh}(t\,,\,t)\} \qquad \text{transit flows}$$

$$A^i(t\,,\,t+k) = \{g^{ai}_{nh}(t\,,\,t+k) \qquad \text{for assets}$$

$$E^i(t\,,\,t-k) = \{g^{ei}_{nh}(t\,,\,t-k)\} \qquad \text{for equities}$$

$$m^{ai}(t) = (\sum_k 1'A^i(t-k\,,\,t))' \qquad \text{for assets}$$

$$m^{ei}(t) = \sum_k E^i(t\,,\,t-k)1 \qquad \text{for equities}$$

$$r^{ai}(t) = \sum_k A^i(t\,,\,t+k)1 \qquad \text{for assets}$$

$$r^{ei}(t) = (\sum_k 1'E^i(t+k\,,\,t))' \qquad \text{for equities}$$

$$k = 1,\ldots,\infty$$

All these arrays are of order N_F, that is, the order of the F-flow matrix, since every stock i is represented by a column and a row of inflows, respectively, outflows, in matrix F. We specifically denote the latter ones by

$w^i(t)$ the i-th column of F(t)

$q^{i'}(t)$ the i-th row of F(t)

and with all these definitions we can now describe the transformation in stock i in period t (for the sake of brevity dropping the time index again):

$$G^i 1 + m^{ei} + r^{ai} = w^i \tag{9a}$$

$$1'G^i + m^{ai'} + r^{ei'} = q^{i'} \tag{9b}$$

that is, *mobilized opening stock plus transiting inflows equal outflows*, respectively *transiting outflows plus residual in the closing stock equal inflows*. These can be regarded as the basic equations of the transformation process in stock i. (Of course, in case of assets m^{ei} and r^{ei}, in case of equities m^{ai} and r^{ai} would be zero vectors as follows from the above definitions.) They account for the fate of inflows: how they pass through the stock becoming parts of various outflows (in the transformation matrix G^i) or rest frozen in the stock waiting for future exit (in r^{ai}, respectively r^{ei}). On the other hand, they account for the origin of outflows: how they have been combined of mobilized parts of the opening stock (m^{ai}, m^{ei}) plus of parts of various inflows within the period (in G^i). The description is complete and symmetric.

We are, however, not interested in any individual stock, we are interested in the economy as a whole. Therefore, we define

$$G = \begin{bmatrix} G^1 & & \\ & G^2 & \\ & & \ddots \\ & & & G^{N_F} \end{bmatrix}$$

$$w = \begin{bmatrix} w^1 \\ w^2 \\ \cdot \\ \cdot \\ \cdot \\ w^{N_F} \end{bmatrix} \qquad q = \begin{bmatrix} q^1 \\ q^2 \\ \cdot \\ \cdot \\ \cdot \\ q^{N_F} \end{bmatrix}$$

$$m_a = \begin{bmatrix} m_a^1 \\ m_a^2 \\ \cdot \\ \cdot \\ \cdot \\ m^{N_F} \end{bmatrix} \qquad m_a^i = m^{ai} \quad \text{if i asset}$$

$$m_a^i = \underline{0} \quad \text{if i equity}$$

and m_e, r_a, r_e in the very same manner. Obviously (9a) and (9b) hold if we drop the i indices but for keeping records straight let it be repeated here

$$G1 + m_e + r_a = w \tag{10a}$$

$$1'G + m_a' + r_e' = q' \tag{10b}$$

as the *transformation equation* for the economy as a whole.

All these arrays are of order $(N_F)^2$. We could say that the economic system described in the previous section has now been

blown-up by substituting an N_F size block of internal transformation for each row, respectively column, of the F-flow matrix. The density would of course be very low, but thinking in terms of practical computation this is all nonsense anyway. If ever a similar, obviously very aggregated, model would come near to a computer, the first thing to do would be to design a procedure that gets rid of the empty blocks. But, here we do not consider computational procedures, we consider principles and notational convenience.

The substantive feature of the system to observe is that although each stock reports on all contacts it has with other stocks (i.e., on all flows that affect the stock), the transformation matrices of the individual stocks seem to remain isolated, independent of each other. (They all form main-diagonal blocks in a matrix, the off-diagonal blocks of which are all zero matrices.) In this sense, we do not seem to have a system of interdependent transformation processes only an ordered set of independent processes. In fact this is not true; the indirect interdependency can be traced. To develop the analytical tools for this, however, requires a lot of technicalities. Therefore, it will be left to the next chapter.

Having dropped the time indices and limited ourselves again to one time period we also seem to have lost the dynamic aspect of the description. This is not true either. The reader should be reminded that the m and r vectors figuring in (10a) and (10b) have been defined as sums of the past, respectively the future, transformations with respect to the time period considered. Once we will have obtained--in Section 3.2--the tools required for tracing interdependencies within the considered period, we shall be able to see the past developing into the future through the present.

3. TOOLS FOR ANALYSIS

3.1 The Stock-Flow Relation

We set out from the basic stock-flow equation in its more sophisticated form as interpreted for the economy as a whole, from equation (8). (Obviously, whatever we do here can be done with equation (4), that holds for a single unit, as well.) Equation (8) is repeated here for the convenience of the reader:

$$s_a' + z_e' + 1'F = (s_e + z_a + F1)' \qquad (8)$$

We define

$$s_e + z_a + F1 = b \qquad (11a)$$

From (8) it follows that b can be interpreted as the vector of the *total turnover* of stocks, either as opening stocks plus inflows (total disposable sources) or as outflows plus closing stocks (total disposed sources). We shall make use of this either-or pair of alternative, symmetric interpretations.

With b we break down equation (8) into two equations and thereby start the *symmetric* handling of the system:

$$s_a' + z_e' + 1'F = b' \qquad (11b)$$

The trouble with (11) is that because of the reversed interpretation of rows, respectively columns, for assets and for equities, the opening stock of assets and the closing stock of equities appear together and vice versa. In order to avoid this inconvenience we shall use two vectors to be called *selectors:*

$1_a = \{a_n\}$ $\quad a_n = 1$ if the n-th stock is an asset

$\quad a_n = 0$ if the n-th stock is an equity

$1_e = \{e_n\}$ $\quad e_n = 1$ if the n-th stock is an equity

$\quad e_n = 0$ if the n-th stock is an asset

Obviously, their order will be that of F, 1_a selects assets and 1_e selects equities, and

$$1_a + 1_e = 1 = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{12}$$

With these selectors we define (with < > indicating a diagonal matrix)

$$b_a = \langle 1_a \rangle b \qquad b_e = \langle 1_e \rangle b$$

$$\bar{F}_{ad} = F\langle 1_a \rangle \qquad \bar{F}_{ac} = \langle 1_a \rangle F$$

$$\bar{F}_{ec} = \langle 1_e \rangle F \qquad \bar{F}_{ed} = F\langle 1_e \rangle$$

It is easy to see that in $\bar{F}_{ad}$ the columns, in b_a and in $\bar{F}_{ac}$ the rows that belong to equities, are empty, while from b_e, $\bar{F}_{ec}$ and $\bar{F}_{ed}$ the asset records have been wiped out. Therefore, we can now further break up (11a) and (11b) into two meaningful parts:

$$s_e + \bar{F}_{ec}1 = b_e \quad \text{and} \quad z_a + \bar{F}_{ac}1 = b_a \tag{13a}$$

$$s'_a + 1'\bar{F}_{ad} = b'_a \quad \text{and} \quad z'_e + 1'\bar{F}_{ed} = b'_e \tag{13b}$$

Now, both assets and equities, both opening and closing stocks have their respective stock-flow equations.

The next step is to divide flows by total turnover from the logical side. (At some time in the future, this simple word "logical" will have to be replaced by a proper discussion of economic interpretation and implications of "input-type" and "output-type" coefficients in this framework. Here, we must be satisfied with taking care of the proper symmetry formally.) So we introduce

$$F_{ec} = \overline{F}_{ec}\langle b\rangle^{-1} = \langle 1_e\rangle F\langle b\rangle^{-1}$$

$$F_{ac} = \overline{F}_{ac}\langle b\rangle^{-1} = \langle 1_a\rangle F\langle b\rangle^{-1}$$

$$F_{ad} = \langle b\rangle^{-1}\overline{F}_{ad} = \langle b\rangle^{-1}F\langle 1_a\rangle$$

$$F_{ed} = \langle b\rangle^{-1}\overline{F}_{ed} = \langle b\rangle^{-1}F\langle 1_e\rangle$$

and substitute these into (13), in the same time transposing (13a):

$$s_e' + b'F_{ec}' = b_e' \quad \text{and} \quad z_a' + b'F_{ac}' = b_a' \tag{14a}$$

$$s_a' + b'F_{ad} = b_a' \quad \text{and} \quad z_e' + b'F_{ed} = b_e' \tag{14b}$$

We are now ready to put together the jig-saw puzzle of assets and equities. We introduce

$$s = s_a + s_e \quad \text{and} \quad z = z_a + z_e$$

and add (14a) with (14b):

$$s' + b'(F'_{ec} + F_{ad}) = b'_e + b'_a = b' \tag{15}$$

and

$$z' + b'(F'_{ac} + F_{ed}) = b'_a + b'_e = b'$$

$$s' = b'[\underline{\underline{1}} - (F'_{ec} + F_{ad})] \tag{16}$$

and

$$z' = b'[\underline{\underline{1}} - (F'_{ac} + F_{ed})] \qquad \underline{\underline{1}} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{bmatrix}$$

for brevity

$$\underline{\underline{1}} - (F'_{ec} + F_{ad}) = M_s$$

$$\underline{\underline{1}} - (F'_{ac} + F_{ed}) = M_z$$

$$s'M_s^{-1} = b' \quad \text{and} \quad z'M_z^{-1} = b' \tag{17}$$

from which

$$s'M_s^{-1}M_z = z' \quad \text{and} \quad z'M_z^{-1}M_s = s' \tag{18}$$

Thus, we have defined matrices that--if they exist--will record the total, direct and indirect, interdependence symmetrically between opening stock, closing stock and turnover of all assets and equities of all economic units.

3.2 The Transformation Process

For the blown-up system describing intra-stock transformations, we have two accounting identities to start with:

$$m_e + r_a + G1 = w \tag{10a}$$

$$m'_a + r'_e + 1'G = q \tag{10b}$$

The situation is almost identical to that of equations (11a) and (11b) in Section 3.1, with two differences. The first is minor. The n-th stock is now represented by the n-th block of rows and n-th block of columns in matrix G, rather than just by the n-th row and column as in matrix F. Therefore, we need new selectors:

$$\hat{1}_a = \{a_{n(i)}\} \qquad a_{n(i)} = 1 \quad \text{if the n-th block belongs to an asset}$$

$$a_{n(i)} = 0 \quad \text{same if equity}$$

$$\hat{1}_e = \{e_{n(i)}\} \qquad e_{n(i)} = 1 \quad \text{if the n-th block belongs to an equity}$$

$$e_{n(i)} = 0 \quad \text{same if asset}$$

where obviously

$$\hat{1}_a + \hat{1}_e = 1 \tag{19}$$

$$i = 1,\ldots,N_F$$

where N_F is the order of the F-flow matrix (that has been blown-up into G) and $a_{n(i)}$, respectively $e_{n(i)}$ is the i-th entry within the n-th block.

Using these selectors and performing the "logical" scaling immediately, we get

$$G_{ec} = \langle \hat{1}_e \rangle G \langle q \rangle^{-1}$$

$$G_{ac} = \langle \hat{1}_a \rangle G \langle q \rangle^{-1}$$

$$G_{ad} = \langle w \rangle^{-1} G \langle \hat{1}_a \rangle$$

$$G_{ed} = \langle w \rangle^{-1} G \langle \hat{1}_e \rangle$$

By breaking-up (10a), respectively (10b), into two parts, substituting the above coefficient matrices and performing the necessary transposition, we get

$$m_e' + q'G_{ec}' = w_e' \quad \text{and} \quad r_a' + q'G_{ac}' = w_a' \tag{20a}$$

$$m_a' + w'G_{ad} = q_a' \quad \text{and} \quad r_e' + f'G_{ed} = q_e' \tag{20b}$$

However, the second and major difference from Section 3.1 has now become transparent. Rather than having one turnover vector, we now have two, representing inflows on the one hand, and outflows on the other. Moreover, they are placed in the four equations in a criss-cross manner. Therefore, we have to consider the following.

Vector w has been defined as a column vector composed of the columns of the flow-matrix F, while vector q has been composed of the rows of the same F. In other words, they have been defined by making use of the so-called vec-operation:

$$w = \text{vec } F \quad \text{and} \quad q = \text{vec } F' \tag{21a}$$

We know that there exists a so-called commutation matrix, to be denoted here by K, with the following properties (if F and therefore K are square matrices, which is our case):

$$\text{vec } F = K \text{ vec } F' \tag{21b}$$

$$K = K' = K^{-1} \tag{21c}$$

and therefore,

$$K \text{ vec } F = \text{vec } F' \tag{21d}$$

Thus, K is a matrix partitioned into square submatrices, such that the ij-th submatrix has a 1 in its ji-th position and zeroes elsewhere. (Originally, it was called "permuted identity matrix".) Therefore, it transforms the vec of a matrix into the

vec of the transposed of the same matrix.[17)]

Returning to our problem, we substitute (21) into (20), thereby getting

$$m_e' + w'KG_{ec}' = w_e' \quad \text{and} \quad r_a' + q'G_{ac}' = w_a' \tag{22a}$$

$$m_a' + w'G_{ad} = q_a' \quad \text{and} \quad r_e' + q'KG_{ed} = q_e' \tag{22b}$$

It now makes sense to introduce

$$m' = m_a' + m_e' \quad \text{and} \quad r' = r_a' + r_e'$$

and to add (22a) with (22b):

$$m' + w'(KG_{ec}' + G_{ad}) = w_e' + q_a' \tag{23}$$

and

$$r' + q'(G_{ac}' + KG_{ed}) = w_a' + q_e'$$

but we still have to deal with the right hand sides. For this purpose

$$w_a' = w'\langle \hat{1}_a \rangle = q'K\langle \hat{1}_a \rangle \tag{24}$$

$$w_e' = w'\langle \hat{1}_e \rangle = q'K\langle \hat{1}_e \rangle$$

$$q_a' = q'\langle \hat{1}_a \rangle = w'K\langle \hat{1}_a \rangle$$

$$q_e' = q'\langle \hat{1}_e \rangle = w'K\langle \hat{1}_e \rangle$$

and the required sums are

$$w_e' + q_a' = w'(\langle \hat{1}_e \rangle + K\langle \hat{1}_a \rangle) \tag{25}$$

and

$$w'_a + q'_e = q'(K\langle \hat{1}_a \rangle + \langle \hat{1}_e \rangle)$$

For brevity, let

$$(\langle \hat{1}_e \rangle + K\langle \hat{1}_a \rangle) = K_m$$

and

$$(K\langle \hat{1}_a \rangle + \langle \hat{1}_e \rangle) = K_r$$

and then we have

$$m' + w'(KG'_{ec} + G_{ad}) = w'K_m \tag{26}$$

and

$$r' + q'(G'_{ac} + KG_{ed}) = q'K_r$$

$$m' = w'[K_m - (KG'_{ec} + G_{ad})] \tag{27}$$

and

$$r' = q'[K_r - (G'_{ac} + KG_{ed})]$$

for brevity

$$K_m - (KG'_{ec} + G_{ad}) = N_m$$

$$K_r - (G'_{ac} + KG_{ed}) = N_r$$

$$m'N_m^{-1} = w' = q'K \tag{28}$$

and

$$r'N_r^{-1} = q' = w'K$$

from which

$$m'N_m^{-1}KN_r = r' \quad \text{and} \quad r'N_r^{-1}KN_m = m' \tag{29}$$

again for brevity

$$N_m^{-1}KN_r = \hat{G}^m$$

$$N_r^{-1}KN_m = \hat{G}^r$$

$$m'\hat{G}^m = r' \quad \text{and} \quad r'\hat{G}^r = m' \tag{30}$$

It is obvious that $\hat{G}^m = (\hat{G}^r)^{-1}$ --if they exist.

We should now recall from Section 2.7 that all these relate to the time period t and further that

$$m_a' = [m_a^{1'}, \ldots, m_a^{N_F'}]$$

of which $m_a^i = \underline{0}$ if i is an equity, but in case of assets

$$m_a^{i'} = m^{ai}(t)' = \sum_k 1'A^i(t - k, t)$$

similarly,

$$r_a^{i'} = r^{ai}(t)' = \sum_k 1'A^i(t, t + k)'$$

and in case of equities

$$m_e^{i'} = m^{ei}(t)' = \sum_k 1'E^i(t, t - k)'$$

$$r_e^{i'} = r^{ei}(t)' = \sum_k 1'E^i(t + k, t)$$

For convenience let us introduce

$$A^m(t\,,\,\tau) = [\mathring{A}^1, \mathring{A}^2, \ldots, \mathring{A}^{N_F}]$$

$$\mathring{A}^i = A^i(t\,,\,\tau) \quad \text{if } i \text{ asset}$$

$$\mathring{A}^i = \underline{\underline{0}} \quad \text{if } i \text{ equity}$$

$$E^r(t\,,\,\tau) = [\mathring{E}^1, \mathring{E}^2, \ldots, \mathring{E}^{N_F}]$$

$$\mathring{E}^i = \underline{\underline{0}} \quad \text{if } i \text{ asset}$$

$$\mathring{E}^i = E^i(t\,,\,\tau) \quad \text{if } i \text{ equity}$$

and

$A^r(t\,,\,\tau)$, $E^m(t\,,\,\tau)$ constructed similarly except that the individual A^i, E^i blocks would be transposed.

With these notations it follows from above that

$$m_a'(t) = \sum_k 1'A^m(t - k\,,\,t) \tag{31}$$

$$r_a'(t) = \sum_k 1'A^r(t\,,\,t + k) \tag{32}$$

$$m_e'(t) = \sum_k 1'E^m(t\,,\,t - k) \tag{33}$$

$$r_e'(t) = \sum_k 1'E^r(t + k\,,\,t) \tag{34}$$

Substituting these into (30), we get

$$[\sum_k 1'A^m(t - k\,,\,t) + \sum_k 1'E^m(t\,,\,t - k)]\hat{G}^m(t) = r'(t) \tag{35a}$$

and

$$[\sum_k 1'A^r(t\,,\,t + k) + \sum_k 1'E^r(t + k\,,\,t)]\hat{G}^r(t) = m'(t) \tag{35b}$$

In this manner, various layers of the past and the future can be related to each other, their impacts studied separately and cumulatively.

3.3 Existence Problems

It is beyond my competence in mathematics to prove the existence or non-existence of the inverses that appeared at the end of the previous two sections. It would be nice if they existed.

The case of Section 3.1 is simpler: the M_s^{-1} and M_z^{-1} inverses would be nice, straightforward Leontief inverses, except for two disturbing facts.

Fact 1. The jig-saw puzzle trick, the twisting around and transposing various blocks of matrix F before arriving at the matrix to be inverted might cause singularity. I must say so because I cannot prove it does not. I do not think, however, that it does. This is of course only intuition based on long experience. If something makes sense economically, then the mathematical solution usually exists. (Sometimes quite surprisingly to mathematicians.)

Fact 2. The so-called no-stock records would probably cause singularity if previously not taken care of. In the corresponding rows, respectively columns, of the coefficient matrices the sum of coefficients would be 1, due to the lack of exogeneous input (opening stock), respectively output (closing stock). Then subtracting these coefficient-matrices from the unit-matrix, we would get zero-sum rows or columns. I say "probably" because on the other hand, precisely the jig-saw puzzle trick might eliminate this problem--something for future consideration as well as the various possibilities of taking care of such records beforehand.

The case in Section 3.2 is further complicated by the presence of the K commutation matrix and the so-called selectors in the matrices to be inverted. Here I have to give up thinking about singularity. Of course, one should not give up considering the possibilities of economic analysis even if some inverses

do not exist. The next section will briefly point to at least one such possibility.

3.4 The Impact of a Selected Factor

Flow-matrix F is a very sophisticated matter. It has innumerable blocks and subblocks with various economic meanings, it describes a large number of various processes.

One might want to separate one set of economic objects or processes to see how they affect the whole system, how they interact with the rest of the system. Just a few examples:

1. Domestic versus linking-flows (F = D + L);
2. Human versus conventional flows and assets;
3. The α blocks versus the rest of the blocks;
4. The issue of money versus the rest of the system;
5. Any particular kind of goods or services versus the rest of the system.

Each of these cases can be handled by breaking up the original matrix into two parts, say $F = F_1 + F_2$. Without going through the whole tiresome job of the jig-saw puzzle game, it can be stated that one would be able to end up instead of (16) with

$$s' = b'[\underline{\underline{1}} - (A + B)] \tag{36}$$

and

$$z' = b'[\underline{\underline{1}} - (\mathring{A} + \mathring{B})]$$

where A and $\mathring{A}$ would be composed of parts of F_1, B and $\mathring{B}$ of F_2. Then

$$s' + b'B = b'(\underline{\underline{1}} - A) \quad \text{and} \quad z' + b'\mathring{B} = b'(\underline{\underline{1}} - \mathring{A}) \tag{37}$$

$$Q = (\underline{\underline{1}} - A)^{-1} \quad \text{and} \quad \mathring{Q} = (\underline{\underline{1}} - \mathring{A})^{-1} \tag{38}$$

$$s'Q + b'B = b' \quad \text{and} \quad z'\mathring{Q} + b'\mathring{B} = b' \tag{39}$$

$$s'Q + b'B = z'\mathring{Q} + b'\mathring{B} \tag{40}$$

$$s'Q + b'(B - \mathring{B}) = z'\mathring{Q} \tag{41}$$

and

$$z'Q + b'(\mathring{B} - B) = s'Q$$

$$s'Q(\underline{1} - \mathring{A}) + b'(B - \mathring{B})(\underline{1} - \mathring{A}) = z \tag{42}$$

and

$$z'\mathring{Q}(\underline{1} - A) + b'(\mathring{B} - B)(\underline{1} - A) = s$$

Naturally, the A - s could not be completely separated from the B - s. But in (41) and (42), only the second term of the left-hand side contains B - s and that term would be zero if F_2 would be zero. Therefore, with some justification the second term could be regarded as the impact of F_2, the factor of the system selected for separate study.

This procedure might be useful anyway, whether the original inverses exist or not. But it may be applied with the special purpose of circumventing the singularity problem. In this case the blocks of F which are responsible for singularity would have to be collected in F_2. Let me add that singularity is not a meaningless technicality, it has its economic interpretation and significance. Therefore, this procedure, with the detailed study of F_2, would in itself contribute to the understanding of the system.

NOTES

1. This obsession may be as far-fetched and exclusive as to define "...the economy as that segment of the total social system which deals primarily with exchange and the institutions of exchange and, by extension, with exchangeables... I do not regard the economy as being bounded primarily by the activities of production and consumption of exchangeables, even though these activities are clearly relevant." (Boulding 1970, pp. 17-18) Well, at least some relevance of production and consumption is not denied.

2. "...rigor, but alas, also mortis..." says one of the most comprehensive critics of the irrelevance of the theory. (Heilbroner 1970, p. 487)

3. Although T. Kuhn's theory of scientific revolutions seems to have caused some confusion among economists as to what should and what should not be considered a revolution in economics. (See, for example, De Vroey 1975, Baumberger 1980, Gruchy et. alia 1980.)

4. For a bird's eye view of post-Keynesian economics, see, for example, Heilbroner 1980.

5. See, for example, Sweezy 1974.

6. "The first important thing is to develop the theory so that it can deal with a larger range of questions than it now does. For instance, it is not possible to pose any monetary questions in the context of an Arrow-Debreu model since, according to that construction, money would have no role and hence would not be viable. Similarly, the theory cannot explain a market in shares,..." (Hahn 1981, p. 130)

7. See Goldberg 1974.

8. This is what T. Scitovsky does in his very joyful "The Joyless Economy". (Scitovsky 1976)

9. See, for example, Polanyi 1944, Helleiner 1956, Douglas 1978, Littlejohn 1977.

10. A few references: Stone 1977, Mallmann 1977, Richter 1977, Hibbert 1977, Lecomber 1978, Eckaus 1980.

11. Chamley 1981, Keyfitz 1981.

12. From Potter 1965, p. 653.

13. Well, she has her answer: "... I am increasingly skeptical about the ability of the para-politicians to make effective decisions." (Krause 1982)

14. I had great hopes to learn much about systems and about how to handle them by getting some insight into general system theory and systems analysis. I dutifully read a few representative studies on the subject (Bertalanffy 1976, Boulding 19 , Quade 1981, Majone 1981) and found them highly stimulating in a general intellectual sense, but obviously being a poor student, I could not derive helpful conclusions for my own problems. The precise definition of the economy as a system and the characteristics of that system remain for me matters for future consideration.

15. See Augustinovics 1965 and 1970. Demographers seem to have similar notions of "pull" and "push" effects. (For example, Alonso 1974)

16. To some extent I had to build my own terminology because of the ambiguity in the literature. I mostly kept myself to what I believe to be the proper terminology of double-entry bookkeeping according to Ijiri 1965. But this implied that I had to deviate from the National Accounts (United Nations 1968) language where the whole dual side is called "liabilities" and the term "equities" does not appear at all.

17. The definition of the *vec* operation and the properties of the commutation matrix can be checked i.a. in Magnus 1979.

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Table 1a. Conventional Stocks
(Illustrative Classification)

	ASSETS
	Financial Assets
AF1	Money (currency, transferable deposits) held
AF2	Accounts receivable
AF3	Others (bills, bonds, capital participation, etc.) held
	Tangible Assets
AT1	Fixed assets (buildings, machinery, etc.)
AT2	Inventories purchased, received
AT3	Inventories produced
	No-Stock Assets
AN1	Consumption
AN2	Others
	EQUITIES
	Property (Proprietorship)
EP1	Funds
EP2	Current income
	Financial Liabilities
EF1	Money (currency, transferable deposits) issued
EF2	Accounts payable
EF3	Others (bills, bonds, capital participation, etc.) issued

Table 1b. Human Stocks
(Illustrative Classification)

	ASSETS
AH1	Residents
AH2	Contribution rendered
	EQUITIES
EH1	Residents
EH2	Contribution received

Table 2a. Conventional Flows
(Selected Examples)

Sym-bol	Flow	Debit (col)	Credit (row)
1	THE SIMPLE MARKET		
11	Buying for cash	AT3	AF1
12	Selling for cash (at cost, no profit)	AF1	AT3
2	PURCHASE, LAGGED PAYMENT		
21	Product arrives, account payable appears	AT2	EF2
22	Payment	EF2	AF1
3	PRODUCTION		
31	Use of materials	AT3	AT2
32	Use of fixed assets (depreciation)	AT3	AT1
33	Use of labour within the unit	AT3	EP2
34	Use of labour employed, immediate payment	AT3	AF1
35	Use of labour employed, lagged payment	AT3	EF2
36	Payment of wages	EF2	AF1
4	SALE OF FINISHED GOODS		
41	Inventory decreased (at standard cost)	AN2	AT3
42	Customer charged (at selling price)	AF2	AN2
43	Profit on sales added to current income (43 = 42 - 41)	AN2	EP2
5	SALE OF LABOUR		
51	Work performed	AF2	EP2
52	Wage received	AF1	AF2
53	Work performed for cash	AF1	EP2
6	CONSUMPTION		
61	Own-account consumption	AN1	AT3
62	Consumption of purchased goods	AN1	AT2
63	Use of fixed assets	AN1	AT1
64	Use of labour employed (household services)	AN1	EF2
65	Total: outlay of current income (= 61 + 62 + 63 + 64)	EP2	AN1
70	HOUSE BURNS DOWN (NO INSURANCE)	EP1	AT1
8	INCOME TRANSFER		
81	Receiving money (welfare, tax, subsidies, etc.)	AF1	EP2
82	Giving money (tax payment, etc.)	EP2	AF1
83	Receiving gifts in kind (machinery)	AT1	EP2
84	Giving gift in kind (machinery)	EP2	AT1
9	THE FINANCIAL SUPERSTRUCTURE		
91	Receiving a loan	AF1	EF3
92	Buying shares, bonds, etc.	AF3	AF1
RW	SAVING	EP2	EP1

Table 2b. Human Flows
(Selected Examples)

Symbol	Flow	Flow replaced	Debit (col)	Credit (row)
CC	CONSUMPTION	65	AH1	AN1
P	PERFORMANCE			
P1	Work within the unit (producing goods)	33	AT3	EH1
P2	Work within the unit (human care)		AN1	EH1
P3	Labour sold, cash payment	53	AF1	EH1
P4	Labour sold, lagged payment	51	AF2	EH1
T	TRANSITION ON LIFE-PATH			
TA	Asset		AH1	AH1
TP	Performance		EH1	EH1
	IMMIGRATION			
II	Asset-performance (whichever is smaller)		AH1	EH1
IA	Contribution received (surplus asset above performance)		AH1	EH2
IP	Contribution rendered (surplus performance above asset)		AH2	EH1
	EMMIGRATION			
EE	Asset-performance (whichever is smaller)		EH1	AH1
EA	Contribution rendered (surplus asset above performance)		AH2	AH1
EP	Contribution received (surplus performance above asset)		EH1	EH2
	DEATH			
DD	Asset-performance (whichever is smaller)		EH1	AH1
DA	Loss (surplus asset above performance)		EP2	AH1
DP	Inheritance (surplus performance above asset)		EH1	EP2

Table 3. Definitions in Terms of g-s

$k = 0,\ldots,\infty$
$m = 0,\ldots,\infty$

	i is an asset	i is an equity
inflow	$f_{ni}(t) = \sum_k \sum_h g^i_{nh}(t, t+k)$	$f_{ih}(t) = \sum_k \sum_n g^i_{nh}(t+k, t)$
outflow	$f_{ih}(t) = \sum_k \sum_n g^i_{nh}(t-k, t)$	$f_{ni}(t) = \sum_k \sum_h g^i_{nh}(t, t-k)$
opening stock	$s^{ai}(t) = \sum_k \sum_m \sum_n \sum_h g^i_{nh}(t-1-k, t+m)$	$s^{ei}(t) = \sum_k \sum_m \sum_n \sum_h g^i_{nh}(t+m, t-1-k)$
closing stock	$z^{ai}(t) = \sum_k \sum_m \sum_n \sum_h g^i_{nh}(t-k, t+1+m)$	$z^{ei}(t) = \sum_k \sum_m \sum_n \sum_h g^i_{nh}(t+1+m, t-k)$
mobilized opening stock	$m^{ai}_h(t) = \sum_k \sum_n g^i_{nh}(t-1-k, t)$	$m^{ei}_n(t) = \sum_k \sum_h g^i_{nh}(t, t-1-k)$
remaining inflow	$r^{ai}_n(t) = \sum_k \sum_h g^i_{nh}(t, t+1+k)$	$r^{ei}_h(t) = \sum_k \sum_n g^i_{nh}(t+1+k, t)$

Figure 1.

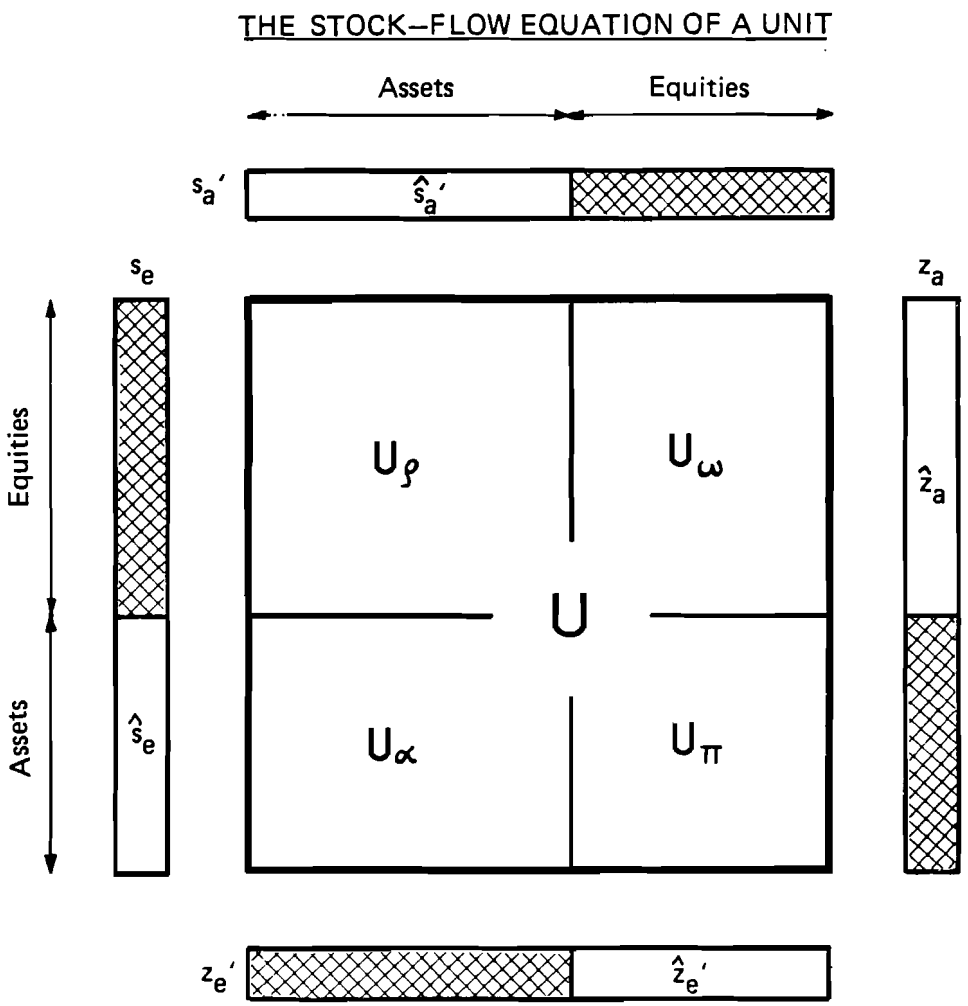

THE STOCK–FLOW EQUATION OF A UNIT
Assets
Equities
s_a'
$\hat{s}_a'$
s_e
z_a
Equities
Assets
U_ρ
U_ω
U
U_α
U_π
$\hat{s}_e$
$\hat{z}_a$
z_e'
$\hat{z}_e'$

Figure 2.

THE CONVENTIONAL FLOW MATRIX OF A UNIT

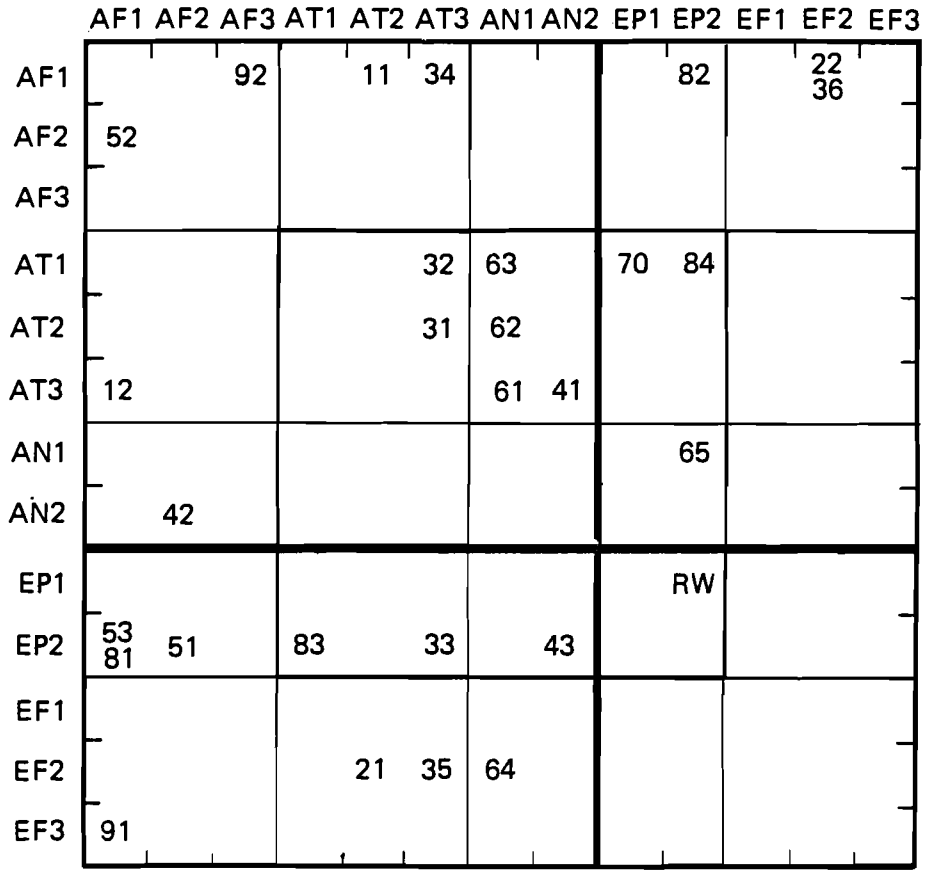


Figure 3.

THE EXTENDED FLOW MATRIX OF A UNIT

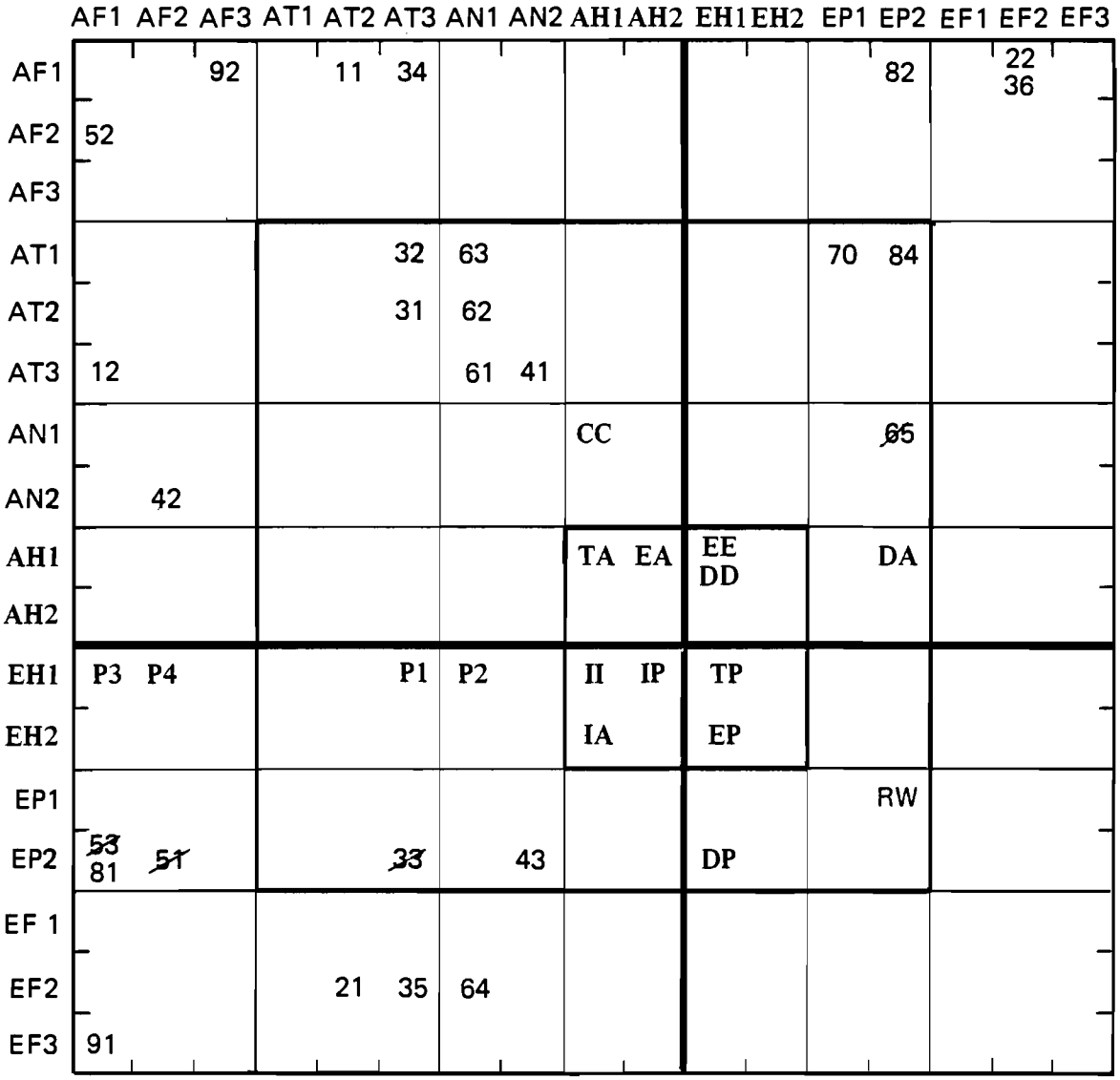


Figure 4.

THE STOCK–FLOW EQUATION OF THE SYSTEM

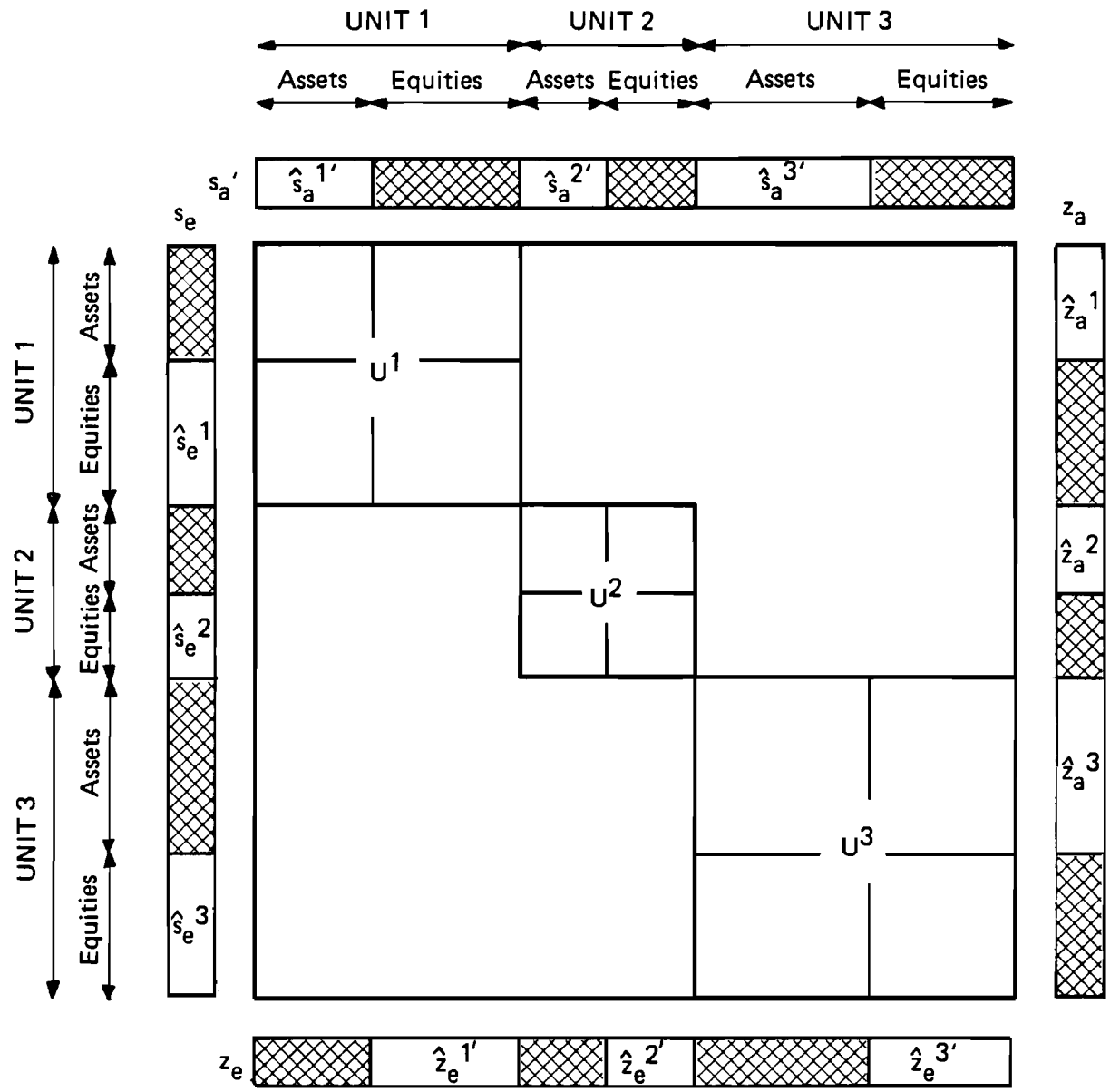


Figure 5.

CONNECTING AND LINKING FLOWS

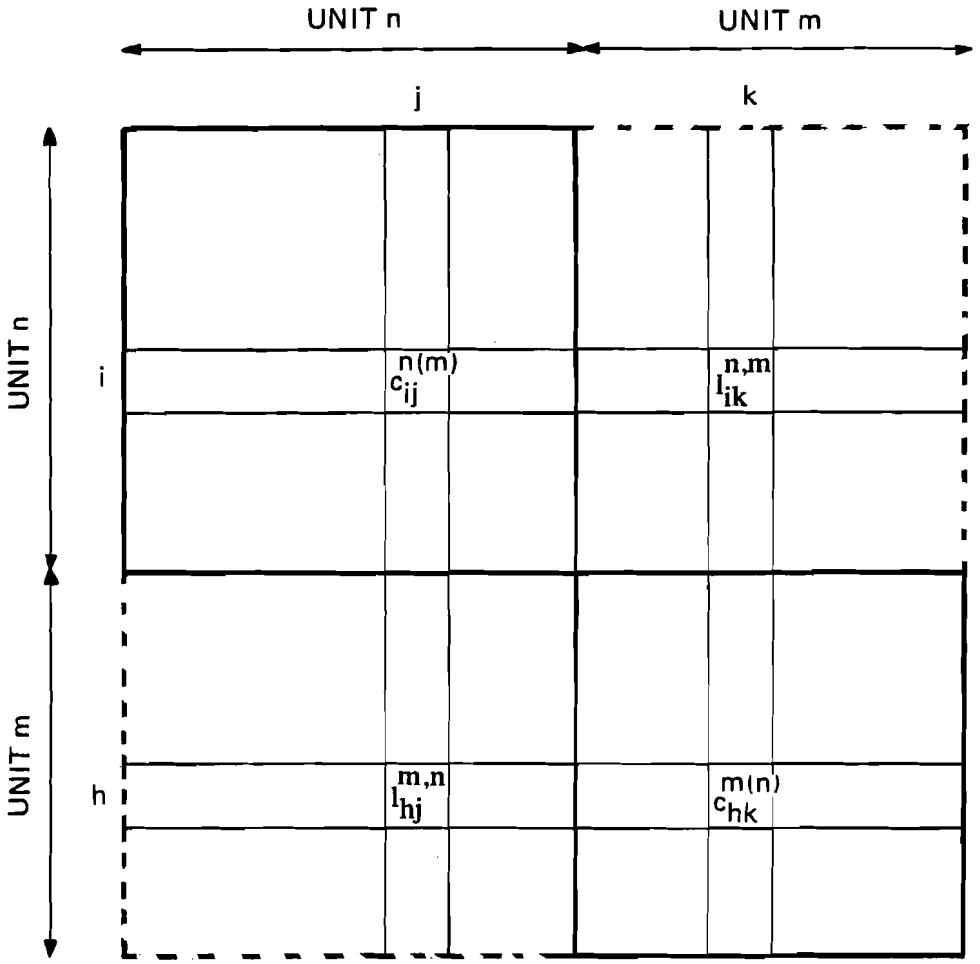


Figure 6.

THE FLOW MATRIX OF THE SYSTEM

D

D^1			
	D^2		
		D^3	
			D^4

L

	$L^{1,2}$	$L^{1,3}$	$L^{1,4}$
$L^{2,1}$		$L^{2,3}$	$L^{2,4}$
$L^{3,1}$	$L^{3,2}$		$L^{3,4}$
$L^{4,1}$	$L^{4,2}$	$L^{4,3}$	

F

D^1	$L^{1,2}$	$L^{1,3}$	$L^{1,4}$
$L^{2,1}$	D^2	$L^{2,3}$	$L^{2,4}$
$L^{3,1}$	$L^{3,2}$	D^3	$L^{3,4}$
$L^{4,1}$	$L^{4,2}$	$L^{4,3}$	D^4

Figure 7.

THE BLOCK–STRUCTURE OF THE FLOW MATRIX

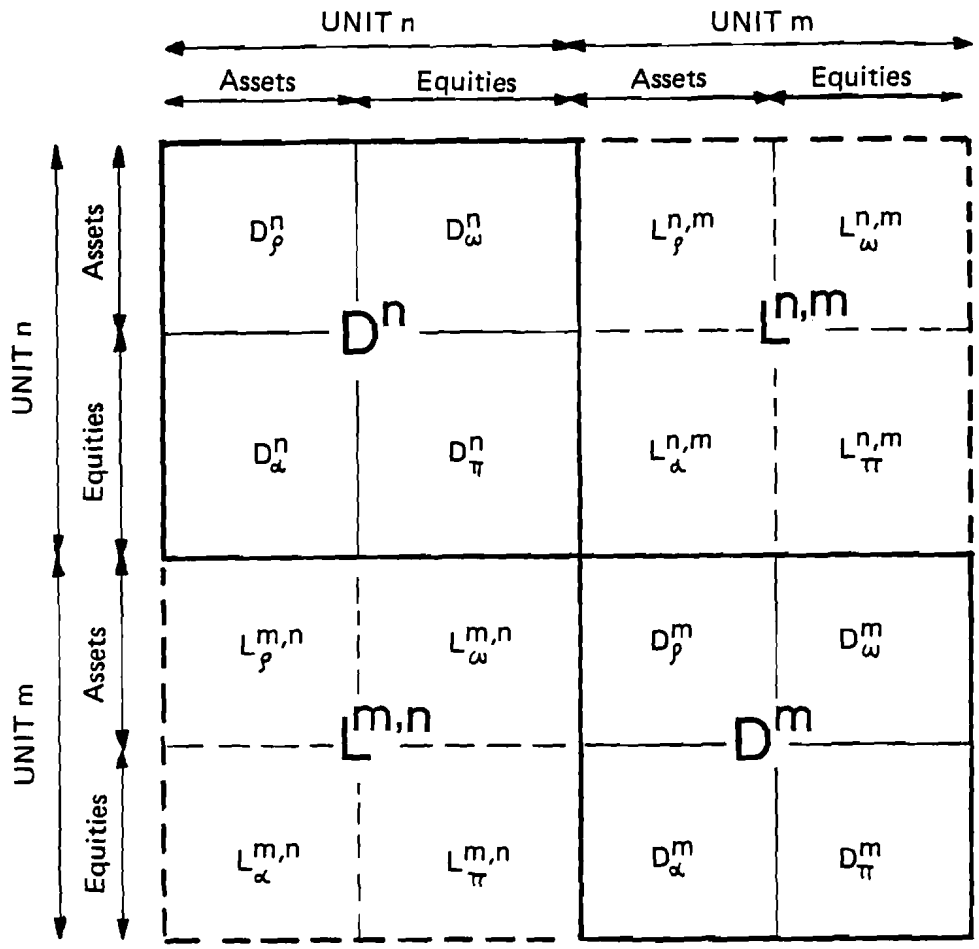


Figure 8a.

INTER–UNIT FLOWS
(COMMODITY–MONEY)

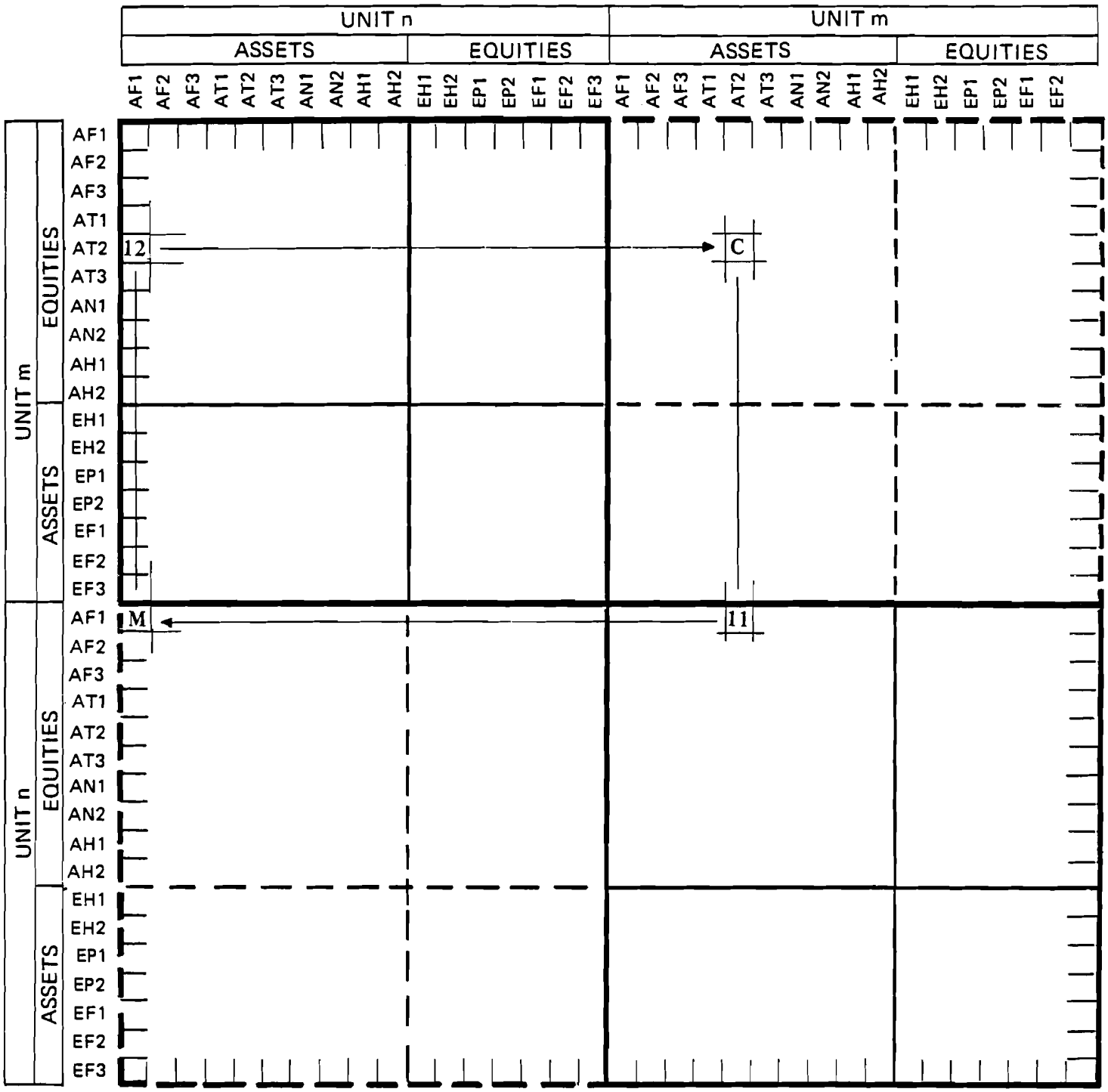


Figure 8b.

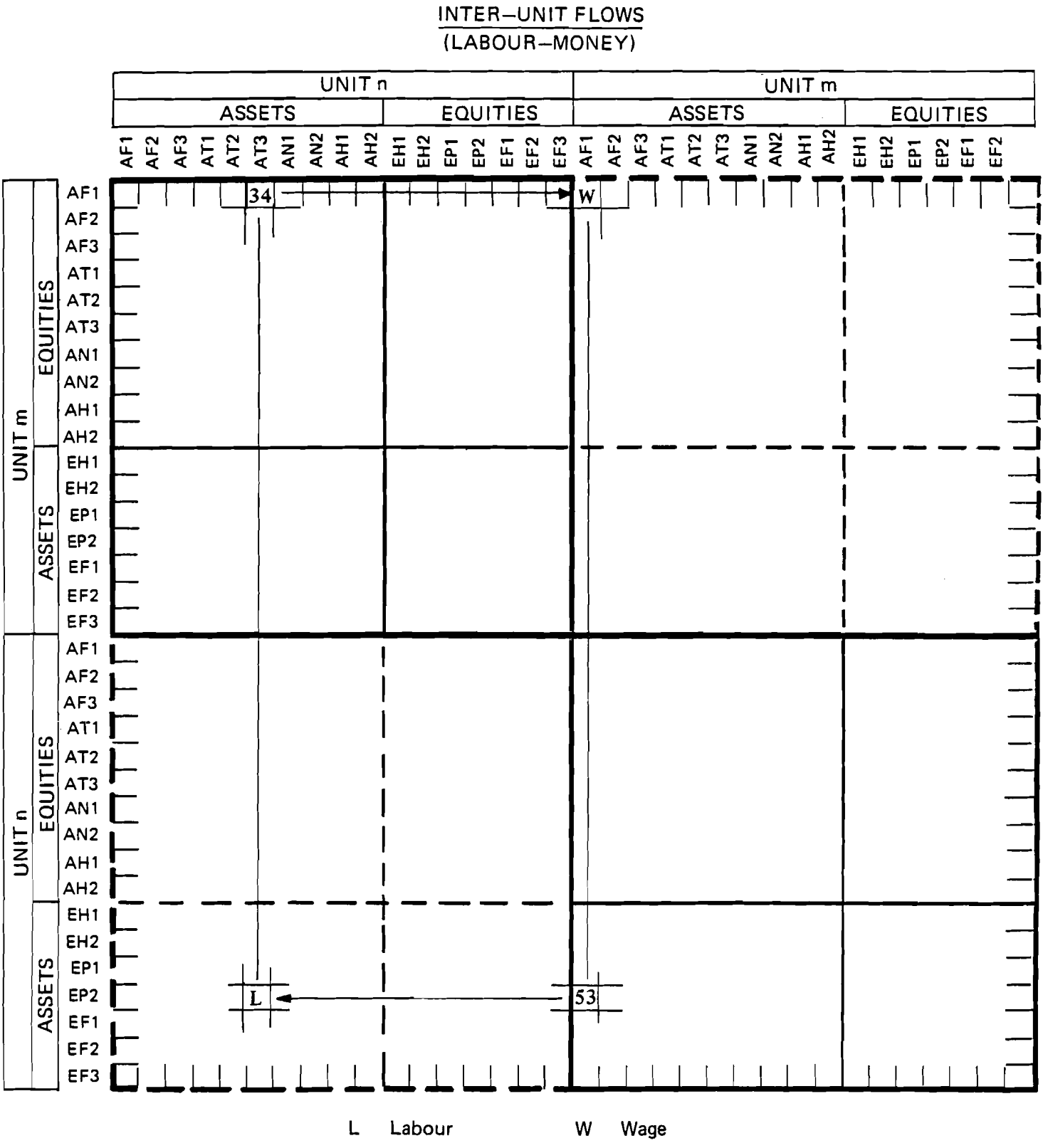
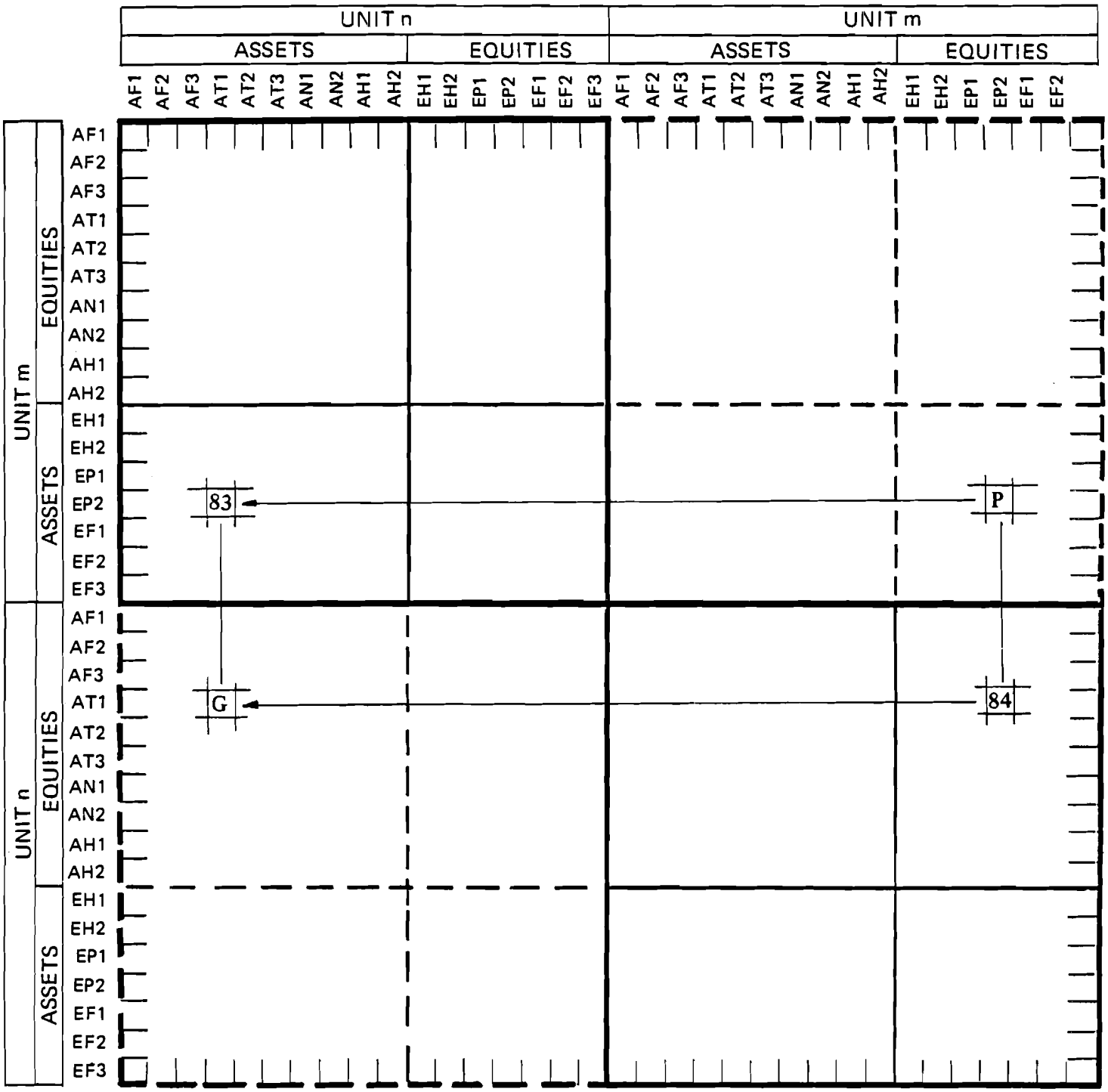

INTER–UNIT FLOWS
(LABOUR–MONEY)
UNIT n
UNIT m
ASSETS
EQUITIES
AF1 AF2 AF3 AT1 AT2 AT3 AN1 AN2 AH1 AH2 EH1 EH2 EP1 EP2 EF1 EF2 EF3
34
W
L
53
L Labour
W Wage

Figure 8c.

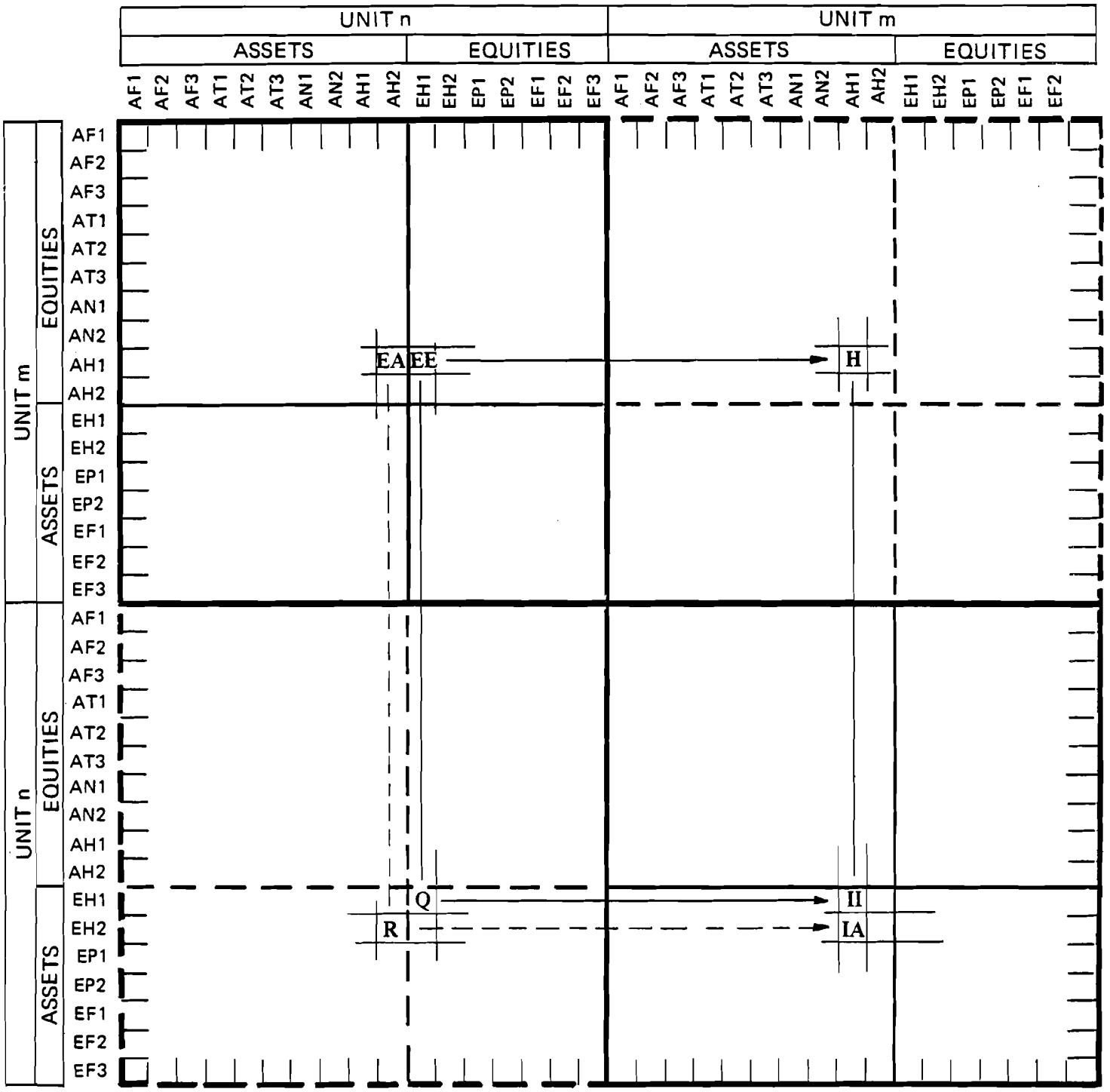
INTER–UNIT FLOWS
(INCOME TRANSFER)



G Gift(Machinery) P Property

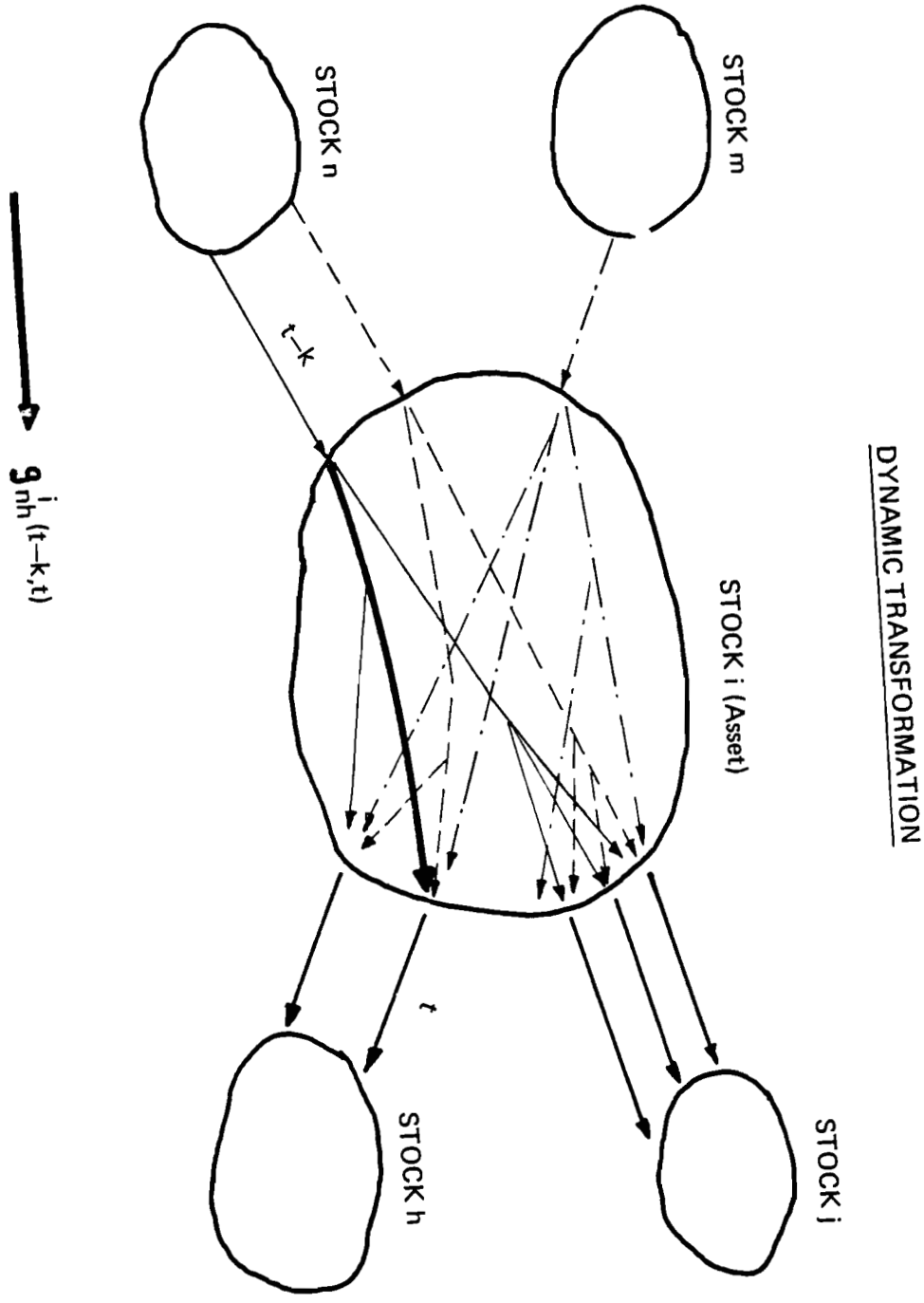
Figure 8d.

INTER–UNIT FLOWS
(HUMAN MIGRATION)



H Residents' Assets Q Residents' Equities R Contribution from n to m

Figure 9.


DYNAMIC TRANSFORMATION
STOCK m
STOCK n
STOCK i (Asset)
STOCK h
STOCK j
t-k
t
$g^{i}_{nh}(t-k,t)$